

中 華 大 學

103學年度研究所碩士班招生入學考試試題紙

系所別：電機工程學系碩士班 組別：通訊、系統、電子電路、光電、微電子暨晶片設計組 科目：工程數學
共2頁 第1頁

* 可攜帶計算機、不可攜帶翻譯機或字典

1. (共15分, 每小題3分) Given Laplace transform $Y(s) = L\{y(t)\} \triangleq \int_0^{\infty} y(t)e^{-st} dt$, please derive

$$(1) L\{1\} \triangleq \int_0^{\infty} e^{-st} dt = \frac{1}{s}$$

$$(2) L\{t\} \triangleq \int_0^{\infty} t e^{-st} dt = \frac{1}{s^2}$$

$$(3) L\{e^{at}\} \triangleq \int_0^{\infty} e^{at} e^{-st} dt = \frac{1}{s-a}$$

$$(4) L\{\cos at\} = \frac{s}{s^2 + a^2} \text{ (Hint: using the previous result with } \cos at = \frac{e^{jat} + e^{-jat}}{2} \text{)}$$

$$(5) L\left\{\frac{dy(t)}{dt}\right\} \triangleq sY(s) - y(0)$$

2. (共20分) Consider the equation $y'(t) + y(t) = t$.

(1) (6分) With the initial condition $y(0) = 0$, solve the equation by using Laplace transform.

(2) (6分) Show that the general solution is of the form: $y = y_h + y_p = C_1 e^{\lambda t} + At + B$.

(3) (7分) Solve the equation by finding a function $v(t)$ so that the equation can be represented as

$$\frac{d}{dt}[v(t)y(t)] = t v(t).$$

3. (共9分, 每小題3分) Given Fourier transform $Y(\omega) = F\{y(t)\} \triangleq \int_{-\infty}^{\infty} y(t)e^{-j\omega t} dt$, please derive

$$(1) F\{e^{-|t|}\} = \frac{2}{\omega^2 + 1} \text{ (Hint: } \int_{-\infty}^{\infty} e^{-|t|} e^{-j\omega t} dt = \int_{-\infty}^0 e^t e^{-j\omega t} dt + \int_0^{\infty} e^{-t} e^{-j\omega t} dt \text{)}$$

$$(2) F\{y(t)\cos at\} = \frac{Y(\omega - a) + Y(\omega + a)}{2}$$

$$(3) F\{y(t-a)\} = e^{-ja\omega} Y(\omega)$$

4. (共6分, 每小題2分) Given Fourier transform $Y(\omega) = F\{y(t)\} \triangleq \int_{-\infty}^{\infty} y(t)e^{-j\omega t} dt$, evaluate (1) $F\{e^{-|t|} \cos 2t\}$,

$$(2) F\{e^{-|t-3|}\}, (3) F\{e^{-|t-3|} \cos(t-3)\}.$$

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5. (10分) Find the general solution to the equation:

$$y'' - 7y' + 10y = 24e^x$$

6. (共10分, 每小題5分) (1) Find the angle between the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{b} = -\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. (2) Find the cross product $\mathbf{a} \times \mathbf{b}$.

7. (共10分, 每小題5分) Consider the vectors $\mathbf{u}_1 = \langle 1, 0, 0 \rangle$, $\mathbf{u}_2 = \langle 1, 1, 0 \rangle$, and $\mathbf{u}_3 = \langle 1, 1, 1 \rangle$ in the vector space R^3 .

(1) Show that \mathbf{u}_1 , \mathbf{u}_2 , and \mathbf{u}_3 are independent.

(2) Express the vector $\mathbf{a} = \langle 3, -4, 8 \rangle$ as a linear combination of \mathbf{u}_1 , \mathbf{u}_2 , and \mathbf{u}_3 .

8. (共20分, 每小題5分) Consider the matrix

$$A = \begin{pmatrix} 5 & -1 & 0 \\ 0 & -5 & 9 \\ 5 & -1 & 0 \end{pmatrix}$$

(1) Find the characteristic equation of A .

(2) Find the eigenvalues of A .

(3) Find the eigenvectors of A .

(4) Find the inverse matrix A^{-1} , if it exists.