中 華 大 學

九十四學年度日間部轉學生入學考試試題紙

系別：電機工程學系　年級：三　科目：工程數學　　共1頁 第1頁

- 本科目可使用計算機

1. Consider the equation

$$(e^x \sin(y) - 2x)dx + (e^x \cos(y) + 1)dy = 0$$

   (a) (5%) Show that this equation is exact.
   (b) (5%) Solve this equation.

2. Consider the equation

$$(x - xy)dx - dy = 0$$

   (a) (5%) Show that $\mu(x) = e^{\frac{x^2}{2}}$ is an integrating factor.
   (b) (5%) Solve this equation by using the above integrating factor.

3. (10%) Solve the Euler's equation $x^2 y'' + 2xy' - 6y = 0$.

4. (10%) Suppose that $U = \begin{bmatrix} a_{11} & 1/\sqrt{2} \\ a_{21} & 1/\sqrt{2} \end{bmatrix}$ is a real unitary matrix, i.e., $U^T U = I$ and $a_{11} > 0$. Find a_{11} and a_{21}.

5. (10%) Let $A = \begin{bmatrix} -1 & 4 \\ 0 & 3 \end{bmatrix}$. Find a matrix P such that $P^{-1}AP = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$. Hint: Consider the eigenvalues and eigenvectors of A.

6. (20%) Given $L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(S)$, derive

   (a) $L\{t\} = \int_0^\infty te^{-st}dt = \dfrac{1}{s^2}$,

   (b) $L\{t\} = \int_0^\infty te^{-st}dt = \dfrac{1}{s^2}$,

   (c) $L\{\sin(t)\} = \int_0^\infty \sin(t)e^{-st}dt = \dfrac{a}{s^2 + a^2}$,

   (d) $L\{\dfrac{df(t)}{dt}\} = \int_0^\infty \dfrac{df(t)}{dt}e^{-st}dt = sF(s) - f(0)$.

7. (10%) Given $\dfrac{df(t)}{dt} + f(t) = t$ and $f(0) = 1$, use Laplace Transform to solve this equation.

8. (20%) Given $F\{g(t)\} = \int_{-\infty}^\infty g(t)e^{-j\omega t}dt = G(\omega)$ and $g(t) * h(t) = \int_{-\infty}^\infty g(\tau)h(t-\tau)d\tau$, derive

   (a) $F\{g(t)e^{jat}\} = \int_{-\infty}^\infty g(t)e^{jat}\, e^{-j\omega t}dt = G(\omega - a)$,

   (b) $F\{tg(t)\} = \int_{-\infty}^\infty t\, g(t)\, e^{-j\omega t}dt = j\dfrac{dG(\omega)}{d\omega}$,

   (c) $F\{g(t) * h(t)\} = \int_{-\infty}^\infty \int_{-\infty}^\infty g(\tau)h(t-\tau)\, d\tau\, e^{-j\omega t}dt = G(\omega)H(\omega)$.

   (d) For $l(t) = \begin{cases} 3, & -1 < t \le 1 \\ 0, & else \end{cases}$, please calculate $F\{l(t)\}$.