＊ 本科目可使用計算機

1. Solve the following differential equations:
   (a) (5%) $(y^2+xy+1)dx+(x^2+xy+1)dy=0$
   (b) (5%) $y''-3y'+2y=e^{2x}+\sin x$
   (c) (10%) $y''+4y'+5y=0;\ y(0)=1,\ y'(0)=-1$

2. The definition of Laplace transform is given as $F(s)=\int_0^{\infty} f(t)e^{-st}dt.$
   (a) (5%) Find the Laplace transform of the function: $f(t)=\int_0^t e^{3\tau}\sin 2\tau\, d\tau$
   (b) (5%) Find the inverse Laplace transform of the function: $F(s)=\dfrac{s+2}{s^2+2s+5}$
   (c) (10%) Using Laplace transform to solve the following initial value problem:
   $y''+4y=5e^t$ with $y(0)=0$ and $y'(0)=0$.

3. (10%) The Fourier series representation is defined as $f(x)=a_0+\sum_{n=1}^{\infty}(a_n\cos\frac{2n\pi}{T}x+b_n\sin\frac{2n\pi}{T}x)$,
   where T is the period of expansion function.
   Find the Fourier series representation of $f(x)$: $f(x)=\begin{cases}0, & 0\le x<\pi\\ 1, & \pi\le x<2\pi\end{cases}$, $f(x)=f(x+2\pi)$.

4. Consider the matrix $\mathbf{A}=\begin{pmatrix}-5 & 9\\ -6 & 10\end{pmatrix}$
   (a) (10 %) Compute $\mathbf{A}^{10}$.
   (b) (10 %) Diagonalizing $\mathbf{A}$.
   (c) (10 %) Find the inverse of $\mathbf{A}$.

5. (10%) Evaluate $\int_C \left(4x^2+4y^2+5z^2\right)^2 ds$ where C is the arc of the circular helix
   $\mathbf{r}(t)=\cos t\hat{\mathbf{i}}+\sin t\hat{\mathbf{j}}+3t\hat{\mathbf{k}}$, from $A:(1,\ 0,\ 0)$ to $B:(-1,\ 0,\ 3\pi)$.

6. (a)(5%) Find the principle values of $(1-i)^{1+i}$; (b)(5%) Evaluate the integral $\int_{\pi i}^{0} z\cos(z)\,dz$.