

1. (20%) Please solve the following equations:
 - (a) $y'' + 9y = \sin 3t$
 - (b) $x^2 y'' - xy' + 4y = x$
2. (20%) Solve the following equations with Laplace Transform.
 - (a) $y'' - 4y = \delta(t)$, $y(0) = 1$, $y'(0) = 0$.
 - (b) $y(t) = t^2 - \int_0^t y(\tau) \cdot (t - \tau) d\tau$.
3. (10%) Write the Laplace transform of $e^{-2t} \sin(3t)$.
4. (20%) Find a real general solution of the linear system:
$$\begin{cases} y_1' = 4y_1 + y_2 - 1 \\ y_2' = 2y_1 + 3y_2 + t \end{cases}$$
5. (20%) Given $f(t) = \begin{cases} 3 & 0 \leq t < 2 \\ 0 & 2 \leq t < 3 \end{cases}$ and $f(t) = f(t + 3)$, derive its Fourier series and complex Fourier series.
6. (10%) Given $f(t) = \begin{cases} 4 & 0 \leq t < 4 \\ 0 & 4 \leq t < \infty \end{cases}$, derive its Fourier Transform $F(\omega)$, where
$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$
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