

可使用計算機*

作答須知：

一、本試卷共**九大題**，請**依題號次序**作答，否則不予計分

二、請力求整齊美觀

一、Find the limits..... 20%

1. $\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$

2. $\lim_{x \rightarrow 0} \frac{\sin 4x}{9x}$

3. $\lim_{n \rightarrow \infty} \sqrt[n]{(3^n + 5^n)}$

4. $\lim_{x \rightarrow 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$

二、Find the derivative of the following functions..... 10%

1. $f(x) = (3x - 2x^2)^3$

2. $f(x) = \left(\frac{3x-1}{x^2+3}\right)^2$

三、Find $\frac{dy}{dx}$ 10%

1. $x^2 - 3 \ln y + y^2 = 10$

2. $x^y = y^x$

四、Find $\frac{dy}{dx}$ of the function $(x+y)^3 = x^3 + y^3$ at $(-1, 1)$ 10%

五、Find the indefinite integrals..... 10%

1. $\int x^2 e^x dx$

2. $x^y = y^x$

(第六大題起，請見次頁)

中 華 大 學

九十六學年度轉學生招生入學考試試題紙

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六、Use a double integral to find the area of the region bounded by $y = x^2$, $y = x^2/8$, and $y = 1/x$, and the area is located **within the first quadrant.** ($\ln 2 = 0.6931$)... 10%

七、Using **Lagrange Multipliers** to find the maximum of $V = xyz$, subject to the constrain $6x + 4y + 3z - 24 = 0$ 10%

八、Solve the equation (**Logistic Growth Function**, hint: to find $y=?$)..... 10%

$$\frac{dy}{dt} = ky(1-y), \text{ Assume } y > 0 \text{ and } 1-y > 0$$

九、The production function for a company is $f(x, y) = 100x^{0.75}y^{0.25}$ where x is the units of labor and y is the units of capital. Suppose that labor unit costs **\$150** and each capital unit costs **\$250**. The total expenses for labor and capital cannot exceed **\$50,000**.

(1) Find the maximum production level.

(2) If **\$70,000** is available for labor and capital, what is the maximum number of units (production level) that can be produced.

(Hint : $\frac{\text{Marginal productivity of labor}}{\text{Marginal productivity of capital}} = \frac{\text{unit price of labor}}{\text{unit price of capital}} = \lambda$)..... 10%