1. Use the Sandwich Theorem to proof that $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ when θ is measured in radians. (10 %)

2. Let f be the function defined as

$$f(x) = \begin{cases} 3 - x, & x < 1 \\ ax^2 + bx, & x \geq 1 \end{cases}$$

where a and b are constants.

(a) If the function is continuous for all x, what is the relationship between a and b? (5 %)

(b) Find the unique values for a and b that will make f both continuous and differentiable. (5 %)

3. Find the tangent and normal line equations for the ellipse $x^2 - xy + y^2 = 7$ at the point (-1, 2). (10 %)

4. (a) Find $\lim_{x \to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$ (5 %); (b)find $\lim_{x \to \infty} \left(1 + \frac{1}{x} \right)^x$ (5 %)

5. Let $f'(x) = 4x^3 - 12x^2$

(a) Identify where the extrema of f occur. (3 %)

(b) Find the intervals on which f is increasing and the intervals on which f is decreasing. (3 %)

(c) Find where the graph of f is concave up and where it is concave down. (4 %)

6. Find the directions in which the function $f(x, y, z) = \ln xy + \ln yz + \ln xz$ increases and decreases most rapidly at (1, 1, 1) then find the derivative of the function in these directions. (10 %)

7. Find the exact length of the curve (10 %).

$$y = \frac{x^3}{3} + x^2 + x + \frac{1}{4x+4} \quad \text{for } 0 \leq x \leq 2$$

8. Find all the local maxima, minima or saddle points, if possible (15%)

$$f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$$

9. Find the volume of the solid region that lies under the paraboloid $z = x^2 + y^2$ and above the triangle enclosed by the lines $y = x$, $x = 0$, and $x + y = 2$ in the xy-plane. (15%)