1. *(15%)* Consider the following problem.

Maximize $Z = 2x_1 + 3x_2$,

subject to

$$x_1 + 2x_2 \le 4$$
$$x_1 + x_2 = 3$$

and $x_1 \ge 0, x_2 \ge 0.$

(1) Solve this problem graphically. （請以圖解法求解）

(2) Using the Big M method, construct the complete first simplex tableau for the simplex method. （請以 大M法 建立起始的 simplex 表格）

2. *(15%)* Consider the following problem.

Maximize $Z = 4x_1 + 3x_2 + x_3 + 2x_4$,

subject to

$$4x_1 + 2x_2 + x_3 + x_4 \le 5$$
$$3x_1 + x_2 + 2x_3 + x_4 \le 4$$

and $x_1 \ge 0, x_2 \ge 0, x_3 \ge 0, x_4 \ge 0.$

Let x_5 and x_6 denote the slack variables for the respective constraints. After you apply the simplex method, a portion of the final simplex tableau is as follows:

Basic Variable	Eq.	Coefficient of:							Right Side
		Z	x_1	x_2	x_3	x_4	x_5	x_6	
Z	(0)	1					1	1	
x_2	(1)	0					1	-1	
x_4	(2)	0					-1	2	

Use the fundamental insight to indentify the missing numbers in the final simplex tableau. Show your calculations. （請完成該 simplex 最終表格，答案卷上請將表格畫出，算法也要寫出）

3. *(10%)* Construct the **dual problem** of the following problem. （不用算出最後答案）

Maximize $Z = x_1 + x_2$,

subject to

$$x_1 + 2x_2 = 10$$
$$2x_1 + x_2 \ge 2$$

and $x_2 \ge 0$ (x_1 unconstrained in sign).

4. **(10%)** 有許多實務上的運輸問題對於運輸的數量有必須為整數的限制，例如，有些產品在工廠出貨時是以棧板為單位，不得打散。 這樣的問題需要以整數規劃來求解嗎？請同時簡要說明其理由。

5. **(14%)** A machine is used to produce precision tools. If the machine is in good condition today, then 90% of the time, it will be in good condition tomorrow. If the machine is in bad condition today, then 75% of the time, it will be in bad condition tomorrow. If the machine is in good condition, it produces 120 tools per day. If the machine is in bad condition, it produces 70 tools per day.
   (1) On the average, how many tools per day are produced?
   (2) Find all mean first passage times.

6. **(15%)** To graduate from Chung Hua University, Grace needs to pass at least one of the three subjects she is taking this semester. She is now enrolled in statistics, OR and Production Control. Grace's busy schedule of extracurricular activities allows her to spend only 4 hours per week on studying Grace's probability of passing each course depends on the number of hours she spends studying for the course (see the following table). Use dynamic programming to determine how many hours per week Grace should spend studying each subject.

Hours of study per week	Probability of Passing Course		
	Statistics	OR	Production Control
0	0.25	0.21	0.15
1	0.28	0.30	0.25
2	0.33	0.36	0.35
3	0.35	0.40	0.42
4	0.40	0.42	0.48

7. **(21%)** A fast-food restaurant has one drive-through window. An average 30 customers per hour arrive at the window. It takes an average of 1.2 minutes to serve a customer. Assume that inter-arrival and service times are exponential.
   (1) On the average, how many customers are waiting in line?
   (2) On the average, how long does a customer spend at the restaurant (from time of arrival to time service is completed)?
   (3) What fraction of the time are more than 2 cars waiting for service (this includes the car (if any) at the window)?