(6%)

1. True or false(是非題)

(a) 在執行卡方適合度檢定時, 每個期望次數 $E_i \geq 5$.

(b) 變異數分析係欲檢定多個母體變異數是否相等之假設檢定.

(c) 以 p 值法進行檢定，當 p 值大於顯著水準則拒絕虛無假設.

(d) 當抽樣之樣本個數越大，越可能拒絕虛無假設.

(e) 母體之標準差越大，估計之誤差越小

(f) 由於樣本平均數可用來估計母體平均數，所以樣本平均數可以大於、小於或等於母體均數

(4%)

2.(單選題) 以 $\hat{\theta}$ 估計 θ 時，當 $\hat{\theta}$ 具有不偏性時，$Var(\hat{\theta})$ 越小是指 $\hat{\theta}$ 越具有下列何種特性？(a) 不偏性　(b) 偏誤　(c) 一致性　(d) 有效性

3.(單選題) 下列何者不是一個良好的估計量應具有的特色？
(a) 不偏性 (b) 有效性 (c) 充分性 (d) 獨立性

(20%)

4. Consider the simple linear regression model:

$$Y_i = \beta_0 + \ln x_i + \varepsilon_i, \quad i = 1,2,\cdots,n$$

Assume that $\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n$ is a random sample from $N(0,\sigma^2)$

(a) Find the least square estimator (最小平方估計量) of β_0

(b) Find $E(\hat{\beta}_0)$

(c) Find $Var(\hat{\beta}_0)$

(d) If the model is to be fitted to the 3 points (1,0), (2,2), (e,1). What is the value of $\hat{\beta}_0$?

(10%)

5. Let $X_1, X_2, \cdots, X_n$ be a random sample from $f_\theta(x) = (1-\theta)\theta^x, x = 0,1,2,\cdots,$ $0 < \theta < 1$. (a) Find the MLE (最大概似估計量) of θ.

(b) Find the sufficient statistic (充分統計量) for θ.

(20%)

6. Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(\mu,\sigma^2)$

(a) Show that the MME(動差法估計量) of (μ,σ^2) is $(\hat{\mu} = \bar{X}, \hat{\sigma}^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n})$.

(b) Show that $\hat{\mu} = \bar{X}$ is consistent(一致的).

(c) Show that $\hat{\sigma}^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n} = \frac{\sum_{i=1}^{n} X_i^2}{n} - \bar{X}^2$ is consistent(一致的).

(15%)

7. Suppose X has the density function $f_\theta(x)=\begin{cases}\frac{1}{\theta} & 0<x<\theta \\ 0 & otherwise\end{cases}$

The hypothesis $H_0:\theta=1$ is to be tested against the alternative $H_1:\theta=2$.

Suppose the critical region(拒絕域) is $C=\{x:x>0.92\}$.

(a) Find the power function $K(\theta)$ of the test.

(b) Find the probability of type I error, α.

(c) Find the probability of type II error, β.

(20%)

8. 在 2002 年，美國公寓的平均月租爲\$895。根據過去資料顯示，母體標準差$\sigma=\$225$。某研究機構要檢定目前美國公寓的平均月租是否超過 2002 年的平均月租，所以隨機抽出 180 個公寓爲樣本，得到樣本平均月租爲\$915，以$\alpha=0.05$檢定目前美國公寓的平均月租是否超過 2002 年的平均月租?

(a) State H_0 and H_1.(寫出虛無與對立假設)

(b) State the decision rule. (寫出決策法則)

(c) Compute the observed test statistic. (計算檢定統計量的值)

(d) What's your conclusion? (結論)

(e) Find the p-value for this test. (求 p 值)

(5%)

9. The lottery commissioner's office in a state wanted to find if the percentage of men and women who play the lottery often are different. A sample of 500 men taken by the commissioner's office showed that 165 of them play the lottery often. Another sample of 300 women showed that 69 of them play the lottery often. Construct a 99% C.I. for the difference p_1-p_2 between the proportions of all men and all women who play the lottery often.(求男女經常玩彩券的比例的差異的 99%的信賴區間)

Table 2 CUMULATIVE NORMAL DISTRIBUTION

$$\Phi(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-t^2/2}\,dt$$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

x	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.891	4.417
$\Phi(x)$	.90	.95	.975	.99	.995	.999	.9995	.99995	.999995
$2[1-\Phi(x)]$	.20	.10	.05	.02	.01	.002	.001	.0001	.00001

TABLE IV

*The t-Distribution**

$$\Pr(T\le t)=\int_{-\infty}^{t}\frac{\Gamma[(r+1)/2]}{\sqrt{\pi r}\,\Gamma(r/2)(1+w^2/r)^{(r+1)/2}}\,dw$$

$$[\Pr(T\le -t)=1-\Pr(T\le t)]$$

	$\Pr(T\le t)$				
r	0.90	0.95	0.975	0.99	0.995
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750

*This table is abridged from Table III of Fisher and Yates; *Statistical Tables for Biological, Agricultural, and Medical Research*, published by Oliver and Boyd, Ltd., Edinburgh, by permission of the authors and publishers.