＊可攜帶計算機＊

(10%)

1. 若 X 為 $[0,1]$ 間的均勻 (uniform) 分配，且 $f_{Y|X}(y|x) = \binom{n}{y} x^y (1-x)^{n-y}$，
$y = 0,1,2,\ldots,n$，求 $E(Y)$。

(10%)

2. 甲、乙兩人獨立地對同一目標射擊一次，其命中率分別為 0.6 和 0.5，現已知目標被命中，求被甲射中之機率。

(15%)

3. 座標軸上每單位代表一個步伐的長度，醉漢在原點起步，左右漫步。假設 (a) 每一步相互獨立 (b) 每一步向右的機率為 p，向左的機率為 $1-p$，隨機變數 X 表示他第 n 步所在的座標，問 (1) 隨機變數 $Y = \dfrac{X+n}{2}$ 的機率分配為何？(2) 他走了 n 步的期望位置(座標值)為何？(3) 若 $p = 0.6$，他走了 10 步，仍停留在左右兩步以內，即 $[-2,2]$，的機率為何？

(15%)

4. 設 X 和 Y 的聯合機率密度函數為 $f(x,y) = \begin{cases} cxy^2, & 0 < x < y < 1 \\ 0, & \text{其他} \end{cases}$，求

(1) $E(X)$ 及 $Var(X)$　(2) $P(\dfrac{1}{4} < X < \dfrac{1}{2} | Y = \dfrac{5}{8})$　(3) $P(2X > Y)$。

(12%)

5. Let the joint pdf of X and Y be $f(x,y) = \begin{cases} e^{-(x+y)} & 0 < x < \infty,\ 0 < y < \infty \\ 0 & otherwise \end{cases}$

(a) Find the pdf of $Z = X + Y$.
(b) Find the joint pdf of $U = X + 2Y$ and $V = X$. (並寫出 u 與 v 的範圍)

(14%)

6. Let X and Y be discrete random variables with joint pmf
$$p(x,y) = \begin{cases} c(x+y) & x = 1,\ 2,\ 3 \text{ and } y = 1,\ 2 \\ 0 & otherwise \end{cases}$$

(a) Find the value c.
(b) Find $E(X)$ and $Var(X)$.
(c) Find $E[(1-X)^2]$.
(d) Find $Var(3 - 2X)$.
(e) Calculate $P(X + Y > 3)$.

帶計算機

(9%)

7. Men arrive at a service counter according to a Poisson at an average of 6 per hour; women arrive at a service counter according to a Poisson at an average of 12 per hour; children arrive at a service counter according to a Poisson at an average of 12 per hour.

(a) What is the probability that at most 3 women arrive at a service counter during the next hour? (下一個小時內，最多 3 個女人到服務台的機率)

(b) What is the probability that no men arrive at a service counter during the next 2 hours? (下二個小時內，沒有男人到服務台的機率)

(c) What is the probability that at least 2 customers (without regard to sex or age) arrive at a service counter in a 5-min period? (在五分鐘期間內，至少 2 個顧客到服務台的機率)

(15%)

8. Suppose X is a random variable with density function

$$f(x) = \begin{cases} 0 & , x < 0 \\ \theta e^{-\theta x} & , x \geq 0 \end{cases} \quad \text{and} \quad P(X \leq 1) = P(X > 1).$$

(a) Find the value of θ.

(b) What is $E(X)$?

(c) Find $E(X^2)$.

(d) Find $E(e^X)$.

(e) Find the hazard rate function $\lambda(t)$.