

1. Solve the Bernoulli differential equation. (15%)

$$x \frac{dy}{dx} - (1+x)y = xy^2$$

2. (a) Given the differential equation $x^2 y'' - 2y = 0$, it is known that $y_1 = \frac{1}{x}$ is a solution. Find the second linearly independent solution, y_2 , using the method of reduction of order. (10%)

(b) If $x^2 y'' - 2y = x^5$, find y_p , the particular solution. (10%)

3. Solve the eigenvalue problem (15%)

$$y'' + (\lambda + 1)y = 0, \quad y'(0) = 0, \quad y'(2) = 0$$

4. Use **Laplace transform** to solve the initial-value problem (16%)

$$y'' + 9y = e^t, \quad y(0) = 0, \quad y'(0) = 0$$

for $y(t)$.

5. For the matrix $\mathbf{A} = \begin{bmatrix} 5 & 6 \\ 3 & -2 \end{bmatrix}$, find a matrix \mathbf{Q} , that diagonalizes \mathbf{A} , and the diagonal matrix \mathbf{D} such that $\mathbf{D} = \mathbf{Q}^{-1} \mathbf{A} \mathbf{Q}$. (16%)

6. Evaluate $\int_C xy^2 ds$ on the quarter-circle C defined by $x = 4 \cos t$, $y = 4 \sin t$, $0 \leq t \leq \pi/2$. (18%)

