

1~15: 單選題(75%) (每題恰有一正確選項, 答對一題得五分, 答錯或不答時, 不倒扣也不給分)

1. What is the domain of the function  $f(x) = x^2 + 2x$  ?

Select the correct answer.

- (A)  $(-\infty, \infty)$  (B)  $[0, \infty)$  (C)  $(-\infty, 0]$  (D)  $(0, \infty)$  (E) none of above.

2. What is the range of the function  $f(x) = x^2 + 2x$  ?

Select the correct answer.

- (A)  $(-\infty, \infty)$  (B)  $[0, \infty)$  (C)  $(-\infty, 0]$  (D)  $(0, \infty)$  (E) none of above.

3. Find the limit,  $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} = ?$

Select the correct answer.

- (A) 0 (B) 1 (C) 0.5 (D) 0.25 (E) none of above.

4. Find the limit,  $\lim_{x \rightarrow \infty} e^{-x} \ln x = ?$

Select the correct answer.

- (A) 0 (B) 1 (C)  $\infty$  (D)  $-\infty$  (E) none of above.

5. Let  $f(x) = \begin{cases} 1 & , x \leq -1 \\ ax + b & , -1 < x \leq 1 \\ 2 & , 1 < x \end{cases}$ . If  $f(x)$  is continuous on  $(-\infty, \infty)$ , then  $a + b = ?$

Select the correct answer.

- (A) 1 (B) 2 (C) 3 (D) 4 (E) none of above.

6. Find  $\frac{d}{dx} x \sin x = ?$

Select the correct answer.

- (A)  $x \cos x$  (B)  $\sin x + \cos x$  (C)  $\sin x + x \cos x$  (D)  $\cos x + x \sin x$  (E) none of above.

7. Find  $\frac{d}{dx} \sin(x^2) = ?$

Select the correct answer.

- (A)  $\cos(x^2)$  (B)  $\cos(2x)$  (C)  $x \cos(x^2)$  (D)  $2x \cos(x^2)$  (E) none of above.

8. If  $f(x) + x[f(x)]^3 = 10$  and  $f(1) = 2$ , find  $f'(1) = ?$

Select the correct answer.

- (A)  $-\frac{16}{13}$  (B)  $-\frac{8}{13}$  (C)  $\frac{16}{13}$  (D)  $-\frac{10}{13}$  (E)  $\frac{10}{13}$

9. Find  $\int_0^{\sqrt{3}} x\sqrt{1+x^2} dx = ?$

Select the correct answer.

- (A) 7 (B)  $\frac{7}{2}$  (C)  $\frac{7}{3}$  (D)  $\frac{7}{4}$  (E) none of above.

10. Find  $\int_1^{\infty} xe^{-x} dx = ?$

Select the correct answer.

- (A) 0 (B) 1 (C)  $e^{-1}$  (D)  $2e^{-1}$  (E) none of above.

11. Find the absolute maximum value of the function  $f(x) = \sqrt{x} - \frac{1}{3}x$  on  $0 \leq x \leq 9$

Select the correct answer.

- (A) 0 (B)  $\frac{4}{9}$  (C)  $\frac{3}{4}$  (D)  $\frac{3}{2}$  (E) none of above.

12. Find the area enclosed by the given curves  $y = x^2 - 4x$  and  $y = 2x - x^2$

Select the correct answer.

- (A) 9 (B) 18 (C) 27 (D) 45 (E) none of above.

13. If  $f(x, y) = \sin^2(mx + ny)$ , find  $f_{xx}(x, y) = ?$

Select the correct answer.

- (A)  $2m^2 \cos(2mx + 2ny)$  (B)  $2mn \sin(mx + ny)$  (C)  $2n^2 \cos(2mx + 2ny)$   
(D)  $2mn \cos(2mx + 2ny)$  (E) none of above.

14. Consider the following two statements.

(1) If  $F'(x) = f(x)$ , then  $\int f(x) dx = F(x) + C$

(2)  $\frac{d}{dx} \int_a^b f(x) dx = f(x)$

Select the correct answer.

- (A) (1) is correct but (2) is incorrect.  
(B) (2) is correct but (1) is incorrect.  
(C) Both (1) and (2) are correct.  
(D) Neither (1) nor (2) is correct.

15. Let  $f(x)$  and  $g(x)$  be two differentiable functions with  $f(x) \geq g(x), x \in \mathfrak{R}$ . Consider the following two statements.

(1)  $\frac{d}{dx} f(x) \geq \frac{d}{dx} g(x), x \in \mathfrak{R}$

(2)  $\int_a^b f(x) dx \geq \int_a^b g(x) dx$ , for every  $a \leq b$

Select the correct answer.

- (A) (1) is correct but (2) is incorrect.  
(B) (2) is correct but (1) is incorrect.  
(C) Both (1) and (2) are correct.  
(D) Neither (1) nor (2) is correct.

**16~20：複選題(25%) (每題至少有二個正確選項, 完全答對得五分, 其餘情形得 0 分)**

16. By definition, if  $f(-x) = -f(x)$ , then  $f(x)$  is called an odd function. Which of the following functions are odd functions?

(A)  $|x|$  (B)  $x^2$  (C)  $\sin x$  (D)  $\tan x$  (E)  $e^x$ .

17. If  $f(x) = \sqrt{x}$  and  $g(x) = \sqrt[3]{1-x}$ , which of the following statements are correct?

- (A)  $f \circ g(-7) = \sqrt{2}$ .  
(B) The domain of  $g \circ f(x)$  is  $(-\infty, \infty)$ .  
(C)  $g \circ f(4) = -1$ .  
(D)  $g \circ g(1) = 0$ .  
(E) The range of  $f \circ f(x)$  is  $(-\infty, \infty)$ .

18. Consider the function  $f(x) = |x|, x \in \mathfrak{R}$ . Which of the following statements are correct?

- (A)  $f(x)$  is continuous on  $\mathfrak{R}$ .  
(B)  $f(x)$  is differentiable for every  $x \in \mathfrak{R}$ .  
(C)  $f(x)$  is an even function.  
(D)  $f(x)$  is an increasing function.  
(E)  $\int f(x) dx = \frac{1}{2}x^2 + C$ .

19. Which of the following conditions are necessary for a function  $f(x)$  being continuous at  $x_0$ ?

- (A)  $\lim_{x \rightarrow x_0} f(x)$  exists.  
(B)  $f(x)$  is differentiable at  $x_0$ .

(C)  $f(x_0)$  is well-defined.

(D)  $\lim_{x \rightarrow x_0} f(x) = f(x_0)$ .

(E)  $\lim_{x \rightarrow x_0^-} f(x) = \lim_{x \rightarrow x_0^+} f(x)$ .

20. Which of the following equalities are correct?

(A)  $\int_0^1 \int_0^1 f(x) + g(y) dx dy = \int_0^1 f(x) dx + \int_0^1 g(y) dy$ .

(B)  $\int_0^1 \int_0^1 f(x) - g(y) dx dy = \int_0^1 f(x) dx - \int_0^1 g(y) dy$ .

(C)  $\int_0^1 \int_0^1 f(x) \cdot g(y) dx dy = \int_0^1 f(x) dx \cdot \int_0^1 g(y) dy$ .

(D)  $\int_0^1 \int_0^1 \frac{f(x)}{g(y)} dx dy = \frac{\int_0^1 f(x) dx}{\int_0^1 g(y) dy}$ .

ANSWER

1	2	3	4	5	6	7	8	9	10
A	E	D	A	B	C	D	B	C	D

11	12	13	14	15
C	A	A	A	B

16	17	18	19	20
C, D	A, C	A, C	A, C, D, E	A, B, C