

可攜帶計算機

作答須知：

- 一、本試卷共八大題，請依題號次序作答，否則不予計分
- 二、請力求整齊美觀

一、True or False?

Determine whether each statement is true or false. If a statement is false, provide an example that shows that the statement is not true in all cases or explain it. (30%)

1. () If **A** and **B** are invertible, the **BA** is invertible.
2. () If the matrices **A**, **B**, and **C** satisfy **AB = AC**, then **B = C**.
3. () For any matrix **C**, the matrix **CC^T** is symmetric.
4. () If the determinant of an $n \times n$ matrix **A** is nonzero, then **AX = 0** has only the trivial solution.
5. () If **A** is an $m \times n$ matrix of rank r , then the dimension of the solution space of **AX=0** is $m-r$.
6. () The system of linear equations **AX=b** is inconsistent if and only if **b** is in the column space of **A**.
7. () The fact that an $n \times n$ matrix **A** has n distinct eigenvalues does not guarantee that **A** is diagonalizable.
8. () If **W** is a subspace of \mathbb{R}^2 , then **W** must contain the vector $(0,0)$.
9. () Let $T: V \rightarrow W$ be a linear transformation from an n -dimensional vector space **V** into a vector space **W**. Then the sum of the dimensions of the range and the kernel is equal to n .
10. () If **A** is a $n \times n$ matrix and c is a nonzero scalar, then the determinant of the matrix **cA** is given by $nc \cdot \det(\mathbf{A})$.

二、Find the indefinite integral : (10%)

(1) $\int \frac{3x^2 + 2x - 1}{x^2} dx$ (2) $\int \frac{1}{1 + e^{-x}} \cdot dx$

三、Find the indefinite integral : (10%)

(1) $\int x^2 \ln x dx$ (2) $\int x^2 e^x dx$

四、Find the definite integral : (10%)

(1) $\int_0^1 x(x+5)^4 dx$ (2) $\int_0^6 \frac{2x}{\sqrt{2x-1}} dx$

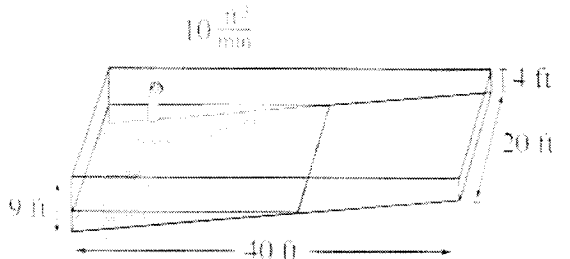
五、Solve the equation (Logistic Growth Function, hint: to find $y=?$) : (10%)

$\frac{dy}{dt} = ky(1-y)$, Assume $y > 0$ and $1-y > 0$

六、Find the minimum value of $f(x, y, z) = x^2 + y^2 + z^2$, subject to the constraints: $x + y - 3 = 0$, $x + z - 5 = 0$ and show the values of x, y, z . (10%)

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七、A Swimming pool is 40 feet long, 20 feet wide, 4 feet deep at the shallow end and 9 feet deep at the deep end (figure2). Water is being pumped into the pool at the rate of 10 cubic feet per minute. How fast is the water level rising when there is 4 feet of water in the deep end?



(10%)

八、Use a double integral to find the area of the region bounded by $y = x^2$, $y = x^2/8$, and

$y = 1/x$, and the area is located within the first quadrant. $\ln 2 = 0.6931$ (10%)