1. Write down the type of particular solution, don't solve it.
   (a) $y'' - 9y' + 14y = 3x - 5\sin 2x + 7xe^{6x}$ (3%)
   (b) $y^{(4)} + y''' = 1 - x^2 e^{-7x}$ (3%)
   (c) $y'' + 4y = (x^2 - 3)\sin 2x$ (5%)
   (d) $y^{(4)} - y'' = 4x + 2xe^{-x}$ (5%)

2. Solve the following ordinary differential equations.
   (a) $e^x yy' = e^{-y} + e^{-2x-y}$. (8%)
   (b) $(1 + \ln x + \frac{y}{x})dx = (1 - \ln x)dy$. (8%)

3.
   (a) For a function f(t), what is the definition of Laplace transform ? (3%)
   (b) Find $L\{e^{2t}(t-1)^2\}$. (5%)
   (c) Find $L\{t\int_0^t 3\tau e^{-\tau} d\tau\}$. (10%)

4. The vectors $\mathbf{u_1} = \langle 1,0,0 \rangle$, $\mathbf{u_2} = \langle 1,1,0 \rangle$, and $\mathbf{u_3} = \langle 1,1,1 \rangle$ form a basis for the vector space R^3.
   (a) Show that $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ are linearly independent. (8%)
   (b) Express the vector $\mathbf{a} = \langle 3,-4,8 \rangle$ as a linear combination of $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$. (7%)

5. Consider the system $\mathbf{Ax} = \mathbf{c}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 1 & 3 & 0 & 2 \\ 1 & 0 & 3 & 3 & -1 & 6 \\ 2 & -1 & 2 & 1 & -1 & 7 \\ 1 & 0 & 5 & 8 & -1 & 7 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 4 \\ 3 \\ 9 \\ 1 \end{bmatrix}.$$

   (a) Use Gauss Elimination to solve this system and determine the rank of **A** and **A**|**c**. (12%)
   (b) Separate the obtained solution of this system into particular solution and homogeneous solution.(8%)

   Note: Clearly indicate every steps of the elementary row operations used in your solution.

6. Evaluate $\oint_C (x^2 + y^2)dx - 2xydy$ on the given closed curve C. (15%)