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子計畫三：利用模糊方法來做非線性隨機系統的多目標追蹤控制(I)

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Control and Estimation for Nonlinear Stochastic Systems

子計畫三：利用模糊方法來做非線性隨機系統的多目標追蹤控制

Multiobjective Robust Tracking Control for Nonlinear Stochastic Systems via Fuzzy Approach

期末成果報告

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一、中文摘要

利用模糊方法來做非線性隨機系統的追蹤控制，此非線性隨機系統包含有界及隨機的不確定性。因為不確定性及干擾只影響系統的動態模式，而不會直接影響其運動模式。亦既，系統的動態模式及運動模式可分別獨立設計其控制器，所以提出一個雙迴路的架構，此架構的內迴路是動態模式，外迴路是運動模式。因此，運動控制器結合韌性動態控制器，並利用適應模糊消去非線性，來處理有界及隨機的不確定性。

關鍵詞：非線性、隨機、模糊。

Abstract

Tracking control using fuzzy method for nonlinear stochastic system consisted of bounded and stochastic uncertainties is studied in the work. Since disturbances and uncertainties can only affect on the dynamical model, their effect can not directly act on the kinematical model. It implies that the dynamical and kinematical models of the original dynamical system can be separated, and each controller can be designed independently. Therefore, a two-loop control scheme is proposed in the work, and its inner-loop is consisted of the dynamical model and its outer-loop contains the kinematical model. Thus, in the work a kinematical controller is integrated with a robust dynamic controller to deal with bounded and stochastic uncertainties with the aid of an adaptive fuzzy elimination scheme which can reduce the nonlinear effect.

Key Words: Nonlinear, stochastic, fuzzy.

二、緣由與目的

A tracking control method through time-varying state feedback based on the back-stepping technique is proposed for a kinematical and the model of two-degree-of-freedom mobile robot [1]. A systematic way to design time-variant feedback control laws requires a class of controller of the nonlinear system. Exponential stabilization of drift less nonlinear control systems uses homogeneous feedback [2]. Stabilization of a nonholonomic system via sliding models has been studied in [3]. External disturbances and parameter uncertainty of a mobile robot driven by two independent wheels have considered in [4]. In the autonomous mobile robot system, the dynamical model is known, but viscous friction is usually consisted of the nominal part and stochastically distributed part. Therefore, a robust controller is recommended for such control design, since it allows simplification of modeling and also considers parameter variation, load change, elasticity of the wheels and road disturbance. The H_∞ control design has been developed to minimize the worst from the disturbances energy point of view. In the H_∞ optimal design [5]-[6], the disturbance is measured by the L_2 norm. Therefore, the external disturbances are constrained to have finite energy (bounded L_2 norms. Most researches normally designed the problem based on one loop which considers dynamical and kinematical models together as one system. However, the kinematical model physically is a pure particle motion unrelated to the mass and force, so it is free of the uncertainties and disturbances. On the other hand, the dynamical model is related to the mass and force, so the robust problem only is inherited in the dynamical model.

A two-loop scheme in [7]-[9] is adopted in the work, but the stochastic uncertainty is additionally considered here instead of the pure bounded uncertainty. In this scheme the dynamical controller is designed in the inner-loop with a suitable bandwidth to migrate the effect of uncertainties and disturbances; and the kinematical controller is designed in the outer-loop for the nonlinear kinematical model of mobile robots. In other words, the proposed two-loop control is to design kinematical control and dynamical control separately then to integrate them together. The proposed scheme can reduce the dimension of dynamical control design, because its target is focused on the dynamical model which is unlike the traditional approaches using the dynamical system consisted of the dynamical and kinematical models. Moreover, Effect of disturbance, bounded and stochastic uncertainties can be reduced by applying the fuzzy method and the stochastic robust H_∞ control design [10].

三、研究方法及成果

1. **Kinematical Model**

The kinematical model of the mobile robot is described and formulated in the section, and it is basically unrelated to the mass, force, uncertainties, and disturbances. The associated dynamical model affected by the uncertainties and disturbances is then discussed in next section to work on the stochastic robust issue.

A. Kinematics of the Mobile Robot

A kinematical model of the front mobile robot possesses two inputs, the center linear and angular velocities v and ω, respectively, and the differential equations describing the kinematics of a mobile robot are given as follows.

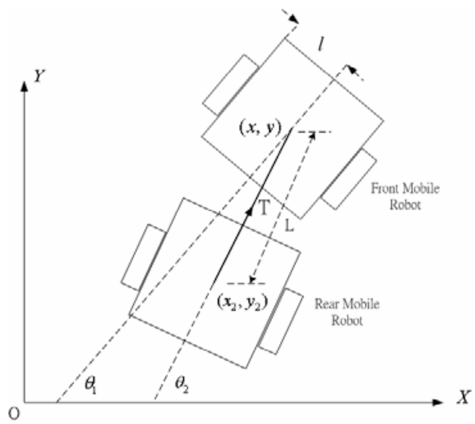

Figure 1.A wheeled mobile robot with trailer

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \omega. \end{aligned} \tag{1}$$

Fig. 1 depicts a wheel mobile robot with trailer in the absolute coordinates. The corresponding difference between target and absolute coordinates is further defined as follows.

$$x_{eo} = x_r - x$$

$$y_{eo} = y_r - y$$

$$\theta_{eo} = \theta_r - \theta$$

where $(x_{eo}, y_{eo}, \theta_{eo})$, (x_r, y_r, θ_r) and (x, y, θ) are error, target and absolute coordinates, respectively. In order to discuss the basis on the mobile robot itself, the following transformation equation is applied.

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{eo} \\ y_{eo} \\ \theta_{eo} \end{bmatrix}, \tag{2}$$

The new error differential equations for kinematical tracking problems of the mobile robot can be reformulated as

$$\begin{aligned} \dot{x}_e &= -v + v_r\cos\theta_e + \omega y_e \\ \dot{y}_e &= v_r\sin\theta_e - \omega x_e \\ \dot{\theta}_e &= \omega_r - \omega. \end{aligned} \tag{3}$$

It notes that as referred from (3), if both of x_e and θ_e equal zero, then we obtain $\dot{y}_e = 0$ (i.e., y_e is constant). This implication can be interpreted that when drivers park a car along the side way, the main trouble is that some offset with the side way (y_e is constant) always happens to drivers. Similarly, it is also the worst case of mobile robot tracking problem, so a good kinematical controller is to make sure that the convergent rate of y_e is the fastest one in order to avoid the above problem. Furthermore, the mobile robot can turn any angle freely with a zero radius, so y_e should be particularly designed to decay to zero before θ_e does for avoiding the worst case. Then the following two Lyapunov functions can meet the requirements.

$L_{k1} = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 + \frac{1}{2}c(\theta_e - \phi)^2$ and

$L_{k2} = \frac{1}{2}\bar{x}_e^2 + \frac{1}{2}y_e^2 + \frac{1}{2}c\theta_e^2$, where $\bar{x}_e = x_e - M\omega y_e$

The first one means that θ_e decays to zero only for $\phi = 0$, and the second one means that x_e decays to zero only when $y_e = 0$. ϕ is chosen to be a function of y_e with the assumption that $y_e = 0 \Rightarrow \phi(0) = 0$. The weighting parameter $0 < c < 1$ is selected for paying less care upon the decay of θ_e and more care on the decay of x_e and y_e.

The following approach is similar to that in [1] with some modifications, i.e., v_d is in term of ω. Three remarks are introduced first.

Remark 1.1: Let a function $\zeta(\theta,\phi)$ be defined as

$$\zeta(\theta,\phi) = \begin{cases} \dfrac{\sin\theta - \sin\phi}{\theta - \phi} & \text{for } \theta \neq \phi \\ \cos\phi & \text{for } \theta = \phi, \end{cases} \tag{4}$$

and it can be extended as a continuous function.

Remark 1.2: Let ϕ be a continuous monotonic decreasing function of $y_e v_r$, and $\phi \in [-\pi/2, \pi/2]$. Then $y_e v_r \sin(\phi) < 0$ if $y_e v_r \neq 0$.

Remark 1.3: In this work the function ϕ is selected as

$$\phi = (\pi/2)\tanh(-a y_e v_r) \tag{5}$$

$$\dot{\phi} = \frac{d\phi}{dt} = \frac{\pi}{2}\text{sech}^2(-ay_e v_r)(-a\dot{y}_e v_r - ay_e \dot{v}_r)$$
$$= \frac{\pi}{2}(1-\tanh^2(-ay_e v_r))(-a\dot{y}_e v_r - ay_e \dot{v}_r) \quad (6)$$
$$= \phi'(-a\dot{y}_e v_r - ay_e \dot{v}_r)$$

where $\phi' = \dfrac{d\phi(-ay_e v_r)}{d(-ay_e v_r)} = \dfrac{\pi}{2}(1-\tanh^2(-ay_e v_r)) = \dfrac{\pi}{2} - \dfrac{2}{\pi}\phi^2$

Theorem 1.1:

For the tracking problem (3) of mobile robots with the Lyapunov function L_{k1}, if the control inputs are designed as

$$\omega_d = \omega_r + \frac{y_e v_r}{c}\zeta + M(\theta_e - \phi) + \phi'(a v_r^2 \sin\theta_e + a y_e \dot{v}_r) \quad (7)$$

$$v_d = v_r \cos\theta_e + Mx_e - c(\theta_e - \phi)\phi' a\omega\, v_r, \quad (8)$$

where $\phi = (\pi/2)\tanh(-ay_e v_r)$, as well as $M > 0$, $0 < c < 1$, and $a > 0$ are constants. Then the system is asymptotically stable as $v_r(t) \neq 0, \forall t \geq 0$.

Remark 1.4: The input v in (9) containing ω is different from that in [1]. The approach can also be interpreted that as driving a car, the direction (related to ω) should be decided before the acceleration does (related to v). In other words, the acceleration somehow needs consider the turning angle (related to ω), so it makes sense that the input v contains ω.

Theorem 1.2:

For the tracking problem of the mobile robot (3) with the given L_{k2}, the corresponding control inputs are designed as

$$\omega_d = \omega_r + \frac{1}{c\theta_e} y_e v_r \sin\theta_e + M\theta_e \quad (9)$$

$$v_d = v_1 - y_e\omega + M\bar{x}_e. \quad (10)$$

Then the system is asymptotically stable.

In the above theorem, it is noted that the system is asymptotically stable by the control inputs (7) (8) and (9) (10) with the adjustable convergence rate M and the weighting factor c.

In practical system, parameters of the dynamical model are varied with some bounded values or even with some stochastic distribution. Bounded uncertainties normally occur on I_w, I_v, M_1 and M_2, and stochastic distribution exists on viscous friction C. The dynamic model of the mobile robot in [5] can be formulated as follows.

$$(M_0 + \Delta M)\dot{q} + (C_0 + \Delta C)q = u + d', \quad (11)$$

where $q = [v\ \omega]^T$, d' is the disturbance, $u = [u_r\ u_l]^T$ are two driving inputs for right and left wheels, and other parameters M_0, ΔM and C_0 can refer to [5]. And the ΔC is considered as a stochastically normal distribution in the work. Therefore, a nominal controller of the dynamic model is designed as

$$u = M_0\dot{q}_d + C_0 q_d + \Delta C q_d + u_0 \quad (12)$$

where u_0 is an auxiliary control input, and $q_d = [v_d, \omega_d]^T$, denotes the desired tracking vector of the linear velocity and angular velocity of the mobile robot.

Remark 1.5: In the work, ΔC is considered as a stochastically normal distribution which has zero mean property. Then the control input in application should be modified as

$$u = M_0\dot{q}_d + C_0 q_d + u_0 \quad (13)$$

To substitute (12) into (11), the systems is a linear system combined with two nonlinear parts $f(\eta)$ and $\bar{d}$, as well as one stochastic part, and it can be written as

$$\dot{e}(t) = Ae(t) + A_1\Delta Ce(t) + u_0(t) + [f(\eta) + \bar{d}(t)], \quad (14)$$

where $f(\eta) = -M_0^{-1}\Delta M\dot{q}$, $\eta = [q, \dot{q}]^T$, $\bar{d} = M_0^{-1}d'$, $A_1 = \text{-}M_0^{-1}$ and the stochastic term ΔC.

Remark 1.6: The dimension of matrix A is 2x2 instead of 4x4 in [5], because the dynamical and kinematical models are separated in this work.

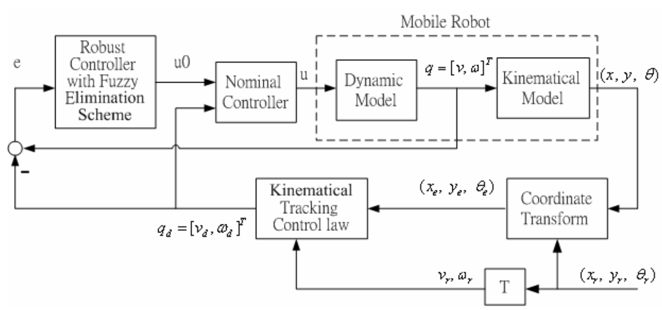

Figure 2.Tracking control system for the mobile robot

In the section, kinematics and dynamic models are integrated together as a two-loop scheme in which the dynamical control is designed in the inner-loop for reducing the effect of uncertainties and disturbances, and the kinematical control is implemented in the outer-loop for kinematical tracking problem, as described in Fig. 2. In the figure, the dynamic and kinematical models are (11) and (1), respectively. The proposed robust controller with fuzzy elimination scheme block is shown in Fig. 2, and its detailed description, as shown in Fig. 3, will be discussed in the next section. The coordinate transformation is (2), and the notation T is denoted to convert from (x_r, y_r, θ_r) to (v_r, ω_r) with the following equations

$$v_r = \sqrt{\dot{x}_r^{\,2} + \dot{y}_r^{\,2}}, \quad \theta_r = \tan^{-1}\left(\frac{\dot{y}_r}{\dot{x}_r}\right), \quad \omega_r = \dot{\theta}_r.$$

Based on the nominal controller (12), the kinematical tracking control law is implemented as (7) (8) or (9) (10).

2. Adaptive Fuzzy Elimination Scheme

The objective of the work is to design a stochastic robust control based on the proposed two-loop control scheme, so the adaptive fuzzy elimination scheme in [4] is directly implemented here with only dimension reduction of the matrix A. An error dynamic system (14) is equivalent to combining the dynamical model and the nominal controller. In section, we will study the adaptive fuzzy elimination scheme.

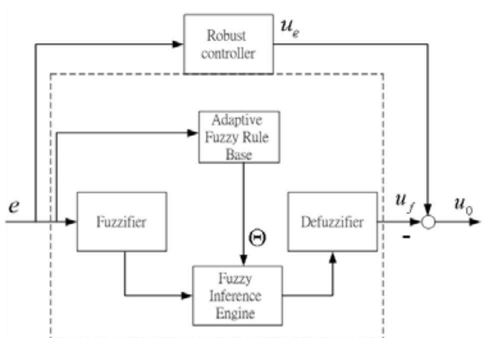

Figure 3.Fuzzy logic systems for wheeled vehicle control

The nonlinear uncertainty $f(\eta)$ of systems is cancelled as much as possible with $u_f(\eta,\Theta)$ by using the fuzzy estimation. With the definition of $u_0 = u_e - u_f(\eta,\Theta)$, and u_e is designed for attenuating the effect of the disturbance $\bar{d}$ and the error $f(\eta) - u_f(\eta,\Theta)$.

$$\dot{e} = Ae + A_1\Delta Ce + u_e + \left[f(\eta) - u_f(\eta,\Theta)\right] + \bar{d} \tag{15}$$

The fuzzy rule base consists of a collection of fuzzy If-Then rules are listed as follows.

$$R^{(l)}: \text{If } \eta_1 \text{ is } F_1^l, \eta_2 \text{ is } F_2^l, \eta_3 \text{ is } F_3^l, \eta_4 \text{ is } F_4^l,$$
$$\text{Then } u_f \text{ is } G^l, \text{ for } l = 1, 2, ..., M \tag{16}$$

The fuzzy basis functions are defined as (17), then the fuzzy logic systems with center-average defuzzifier, product inference and singleton fuzzifier for mobile robots are (18).

$$\xi_{il}(\eta) = \frac{\prod_{j=1}^{4} \mu_{F_j^{il}}(\eta_j)}{\sum_{k=1}^{M}\left[\prod_{j=1}^{4} \mu_{F_j^{ik}}(\eta_j)\right]} \tag{17}$$

$$u_f(\eta,\Theta) = \begin{bmatrix} u_{f_1}(\eta,\Theta) \\ u_{f_2}(\eta,\Theta) \end{bmatrix} = \begin{bmatrix} \xi_1^T\Theta_1 \\ \xi_2^T\Theta_2 \end{bmatrix} = \Xi(\eta)\Theta \tag{18}$$

where $\Theta = \begin{bmatrix} \Theta_1 \\ \Theta_2 \end{bmatrix}$ and $\Xi(\eta) = \begin{bmatrix} \xi_1^T(\eta) & 0 \\ 0 & \xi_2^T(\eta) \end{bmatrix}$;

$\Theta_i = [\theta_{i1} \cdots \theta_{iM}]^T$, $\xi_i(\eta) = [\xi_{i1}(\eta) \cdots \xi_{iM}(\eta)]^T$ for $i = 1,2$, and θ_{il} is singleton.

Let us define the following optimal update parameter of the elimination:

$$\Theta^* = \arg\min_{\Theta}\max_{\eta} \| f(\eta) - \Xi(\eta)\Theta \|.$$

Then, (14) can be rewritten as follows:

$$\begin{aligned}\dot{e} &= Ae + A_1\Delta Ce + u_e + \left[\Xi(\eta)\Theta^* - \Xi(\eta)\Theta\right] + \varepsilon + \bar{d} \\ &= Ae + A_1\Delta Ce + u_e + \Xi(\eta)\tilde{\Theta} + d,\end{aligned}$$

where $\tilde{\Theta} = \Theta^* - \Theta$, $\varepsilon = f(\eta) - \Xi(\eta)\Theta^*$, and $d = \varepsilon + \bar{d}$.

3. Stochastic Robust H_∞ Control

The bounded uncertain of mobile robots is reduced by the above fuzzy approach, and in the section stochastic robust control methods are introduced. Consider the following stochastic system [10]

$$\begin{aligned}de(t) &= (Ae(t) + u_e(t) + d(t))dt + A_1 e(t)\Delta C dt + \Xi(\eta)\tilde{\Theta}dt \\ &= ((A+K)e(t) + d(t))dt + A_1\Delta Ce(t)dw + \Xi(\eta)\tilde{\Theta}dt \end{aligned} \tag{19}$$

where $e(0) = 0$, $e(t) \in R^n$, $u_e(t) = Ke(t) \in R^m$, $d(t) \in R^m$, and $K \in R^{m\times n}$ denote the initial error state vector, control input, external disturbance, and feedback gain matrix, respectively.

For robust control, many methods can achieve the desired performance. The notation and definition are listed, and the robust H_∞ control is studied first.

Notation and Definition

$$I. \| x(t) \|_2^2 = E\int_0^\infty x(t)^T x(t)\, dt$$

$$II. \| x(t) \|_p^2 = \lim_{T\to\infty}\frac{1}{T}E\int_{-T}^{T} x(t)^T x(t)\, dt$$

The H_∞ performance control with the LMI approach is to make the ratio of the error state to the disturbance to be smaller than an upper bound, so it is suboptimal. The disturbance $d(t)$ and error state $e(t)$ are measured by the L_2 norm (or power norm). Therefore, the external disturbances are constrained to have finite energy (or power), i.e., bounded L_2 (or power) norms. Let us consider the following H_∞ performance (20), denote $\|L\|_\infty$ [10], which has been derived to be (20) in [5].

$$\|L\|_\infty = \sup_{\substack{d\in L_F^2(R_+,R^{n_v})\\ d\neq 0, e_0=0}} \frac{\left\{E\int_0^\infty (e^T e + u_e^T u_e)dt\right\}^{1/2}}{\left\{E\int_0^\infty d^T d dt\right\}^{1/2}} < \rho \quad (20)$$

where $\rho > 0$.

The optimal H_∞ control is to minimize (20), so the below theorem is implemented to solve the robust tracking problem [4]. Because the stable matrix A can be selected by the designer, the following inequality (21) is a simplified version without a free adjustable matrix P. A coupling matrix N_c will be derived from the integration of the dynamical and kinematical loops, and it is corresponding to differences between the desired inputs (v_d and ω_d) and the real inputs (v and ω). These differences can be considered in designing the dynamical controller as uncertainty effect to be reduced to desired level $\rho > 0$.

Theorem 3.1:

Consider the system (19) with zero initial conditions, and the following inequality is satisfied

$$\bar{A} + \bar{A}^T + \Delta C^T A_1^T A_1 \Delta C + N_c + \frac{1}{\rho^2} I + K^T K < 0, \quad (21)$$

where $\bar{A} = A + K$ and $N_c = diag\{1/4M, c/4M\}$.

If the adaptive law with fuzzy elimination scheme is designed as follows.

$$\dot{\Theta} = \gamma\, \Xi^T(\eta) e(t) \quad (22)$$

then the system is satisfying the H_∞ performance (20).

Remark 3.1: As we observe the matrix inequality (21), there exists an extra term $\Delta C^T A_1^T A_1 \Delta C$ associated with the stochastic uncertainty part. It indicates that the stochastic uncertainty is mitigated by designing the closed-loop pole of system matrix $\bar{A}$ to be further left in the left-hand plane.

In order to convert (21) into the LMI form, we substitute $\bar{A} = A + K$ into (21) to get

$$(A+K) + (A+K)^T + \Delta C^T A_1^T A_1 \Delta C + N_c + \frac{1}{\rho^2} I + K^T K < 0 \quad (23)$$

By the Schur-complement, it can be formulated as

$$\begin{bmatrix} A + K + A^T + K^T + \Delta C^T A_1^T A_1 \Delta C + \frac{1}{\rho^2} I + N_c & K^T \\ K & -I \end{bmatrix} < 0. \quad (24)$$

Corollary 3.1:

The stochastic robust H_∞ control problem is to solve the following LMI.

Solve K

Subject to

$$\begin{bmatrix} A + K + A^T + K^T + \Delta C^T A_1^T A_1 \Delta C + \frac{1}{\rho^2} I + N_c & K^T \\ K & -I \end{bmatrix} < 0. \quad (25)$$

四、結論與討論

A two-loop control scheme, integrated the kinematical and dynamical controls together, is proposed in the work, especially, the bounded and stochastic uncertainties are considered to design two distinct kinematical controllers which can individually integrate with the same stochastic robust H_∞ control with adaptive fuzzy elimination scheme dynamical controller. The coupling matrix (the difference between desired and real inputs) is considered as an uncertainty in designing the corresponding dynamical controller. The dimension of the dynamical controller design based on the proposed scheme can be reduced. Moreover, disturbance in the inner-loop can be suppressed to a desired level first, and it can further be smoothed in the outer-loop.

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