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非線性隨機控制理論研究及其在非傳統控制領域的應用

子計畫一：非線性隨機建模研究及其在射箭之應用(I)

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摘要

A mathematical model that can more accurately reflect the typical archery style is studied in the paper. Twelve archers from the archery team of National College of Physical Education and Sports were invited to attend this study, and each of their aiming trajectories during the last 1.5 second before releasing the arrow was recorded. Based on their time series of the aiming trajectory, we propose a linear time invariant auto-regressive (AR) process to model them. The second order of the AR model is formulated based on the recorded data of three different time periods scheduled as the last 0.5 second, one second and 1.5 second periods before releasing the arrow. The fitting errors of the AR2 model associated with three time period are also calculated to indicate the fairness of the proposed method. The aiming style may involve a constant offset term in the AR model, so the term is also added to check whether the archer has this special trend or not. The study shows that some archers have the constant offset trend, but some archers do not have it. Seven out of the twelve archers with the their best fitting error of the AR2 model are fallen into the 1.5 second period, as expected. Moreover, the archers with good performance usually fallen into the extreme cases related to the offset term added in the AR2 model.

1. Introduction

Lots of archery researches have been conducted from different approaches in order to find the key point for improving the performance of this fine and highly skilled sport. Lateral deflection of archery arrows. A whole mechanical and mathematical model of an arrow-bow motion system, which accounts for arrow deflection in the lateral plane, has been created (Zanevskyy, 2001). The model takes into consideration the mechanical properties of a string, bow limbs and a grip as an oscillator of concentrated elastic and inertial elements connected with the feathered end of the arrow. Theoretical investigation of natural modes and frequencies of bow and arrow vibration has been conducted. Data for the first four natural frequencies and modes have been obtained and practical conclusions have been drawn. As a result of modeling and computer simulations, an engineering method for matching bow and arrow parameters has been proposed. Comparative results for the wrong and right combination of these parameter values for the modern sport bow and two arrows are presented.

The aiming stability is the key factor affects the archery performance has been indicated by Shiang et al. (1997), and it can be determined by the size of aiming locus. They further pointed out that the aiming locus pattern is also a useful index to determine the performance. The United States Olympic Committee (1996) has defined archery fundamentals as: stringing the bow, stance, nock, set, pre-draw, draw, anchor, aim, release, and follow through. Among these fundamentals, the aim stage will also be analyzed in this paper. Furthermore, analysis of correlation between the aiming adjustment trajectory and the shot points has been studied (Lin et al., 2003). In that paper, fifteen analysis units, in which each analysis unit is the average of six sampling data, are generated for analysis of vertical and horizontal deflections.

In the paper, a mathematical model is proposed to represent the aiming trajectory. That is, we propose a popular linear time invariant AR process (Ljung, 1999) that can model the recorded time series of the aiming trajectory. Individual AR model is established for the aiming trajectory along the vertical and the horizontal directions. We consider the second order of the AR model based on three different time periods. The fitting errors of the AR2 model based on three different time periods along both of the vertical and the horizontal deviations are evaluated. For most of the attendances, the longest one is the best period to obtain the better accuracy of the AR2 model as expected. The aiming style may involve a constant offset term in the AR model, so the constant term is added to check whether the archer has this special trend or not. The study shows that some archers have this kind of trend, but some archers do not have it. Similarly, the suitable time period often occurs at the 1.5 second period for most of them when the constant offset term is included.

2. Methods

Twelve archers from the man archery team of National College of Physical Education and Sports attended this experiment. In our experimental setting, a laser pen is mounted at the bow handle for capturing the aiming trajectory by using a digital video camera. Using the laser pen and the digital camera, both the vertical and horizontal aiming trajectory coordinates during a suitable period before releasing the arrow can be accurately recorded. On the other hand, the vertical and horizontal shot coordinates are captured by another camera placed in front of the target. These recorded data are then processed by APAS (Ariel Performance Analysis System) motion analysis system for studying the aiming procedure and the shot points along the vertical and the horizontal directions. The frequency of the digital video camera is set as 60 fields/second for capturing the light point trajectory, so in the fix field mode 90 fields will be recorded during the one and half second before release of the arrow.

2.1. *Field setup*

The distance between start line and arrow target is 70 meters. The experiment proceeds according to the usual competition procedure. Before the test, they can shoot three arrows to adjust their bow sight. Then the test begins. Each one of them shoots thirty six arrows, that is, 3 arrows will be shot for a round and totally 12 rounds are performed. Each arrow is required to be released within 40 seconds, otherwise it is not recorded.

3. Results and Discussion

The proposed linear time-invariant AR2 model is formulated as follows.

$$\begin{aligned} x(k) &= a_{x1}x(k-1) + a_{x2}x(k-2) + e(k-2) \\ y(k) &= a_{y1}y(k-1) + a_{y2}y(k-2) + e(k-2) \end{aligned} \qquad (1)$$

where $x(k)$ and $y(k)$ are the time series of the horizontal and vertical deviations of the aiming trajectory; a_{x1}, a_{x2}, a_{y1} and a_{y2} are the corresponding coefficients of the AR2 model; $e(k)$ are the white noise with zero mean.

In order to estimate the coefficients of a_{x1}, a_{x2}, a_{y1} and a_{y2}, we form the following vectors and matrices such as

$\theta_x = [a_{x1}\ a_{x2}]^T$, $\theta_y = [a_{y1}\ a_{y2}]^T$, $E(k)) = [e(k-2)\, e(k-1) \ldots e(k-2+m)]^T$

$A_x = [X(k-1) X(k-2)]$, $A_y = [Y(k-1) Y(k-2)]$

where $X(k) = [x(k)\ x(k+1) \cdots x(k+m)]^T$, $Y(k) = [y(k)\ y(k+1) \cdots y(k+m)]^T$ and m is the total number of applied time series. Then, Eq. (1) can be rewritten as

$$\begin{aligned} X(k) &= A_x\theta_x + E(k) \\ Y(k) &= A_y\theta_y + E(k) \end{aligned} \qquad (2)$$

Moreover, the estimation of θ_x and θ_y denoted as $\hat{\theta}_x$ and $\hat{\theta}_y$ can be obtained by

$$\begin{aligned} \hat{\theta}_x &= (A_x{}^T A_x)^{-1} A_x{}^T X(k) \\ \hat{\theta}_y &= (A_y{}^T A_y)^{-1} A_y{}^T Y(k) \end{aligned} \qquad (3)$$

The estimation errors $E_x = (X(k) - A_x\hat{\theta}_x)^T (X(k) - A_x\hat{\theta}_x)/(m+1)$ and $E_y = (Y(k) - A_y\hat{\theta}_y)^T (Y(k) - A_y\hat{\theta}_y)/(m+1)$ are evaluated and listed in Table 1. In order to check whether the offset effect exists in their aiming trajectory, we add the constant terms b_x and b_y into Eq. (1) as

$$\begin{aligned} x(k) &= a_{x1}x(k-1) + a_{x2}x(k-2) + b_x + e(k-2) \\ y(k) &= a_{y1}y(k-1) + a_{y2}y(k-2) + b_y + e(k-2) \end{aligned} \qquad (3)$$

By applying the similar technique as the above, the associated estimation errors with the offset are also calculated and listed in Table 2. Additionally, the estimated constant terms b_x and b_y are also shown in Table 3.

As compared the fitting errors between Tables 1 and 2, the fitting errors by taking the offset into account are better than those without considering the offset, and this result is as expected due to the additional term for data fitting. By considering the fitting error of both the vertical and horizontal deviations together, there are seven archers (archers 1, 2, 7, 8, 9, 11, and 12) whose better fitting errors are located at the 1.5 second period, but for archers 3, 4, and 6, theirs are fallen into the one second period, and the best fitting errors for archers 5 and 10 exist at the half second period.

Table 3 shows that in the one and half second period the offset b_x is insignificant for archer 1 along the horizontal deviation, and it is significant for archer 5; along the vertical deviation, the offset b_y is insignificant for archer 10 but it is significant for archer 12. Similarly, along the horizontal deviation in the one second period the offset b_x is also insignificant for archer 1, and it is significant for archer 5; the offset b_y is insignificant for archer 3 and is also significant for archer 12 along the vertical deviation. Finally, along the horizontal deviation in the half second period, the offset b_x is also insignificant for both archers 12 and 1, and it is significant for archer 10; the offset b_y is also significant for archer 12 but insignificant for archer 5 along the vertical deviation. There are some interesting findings that archers 1, 5 and 12 ranking among the top four are often fallen into these extreme cases.

3.1. *Tables*

Table 1. The fitting errors of the proposed AR2 model without the offset

Period	Direction	Archer 1	Archer 2	Archer 3	Archer 4	Archer 5	Archer 6
1.5 second	Horizontal	0.27139	0.33798	0.28236	0.35735	0.21325	0.32211
	Vertical	0.37681	0.44707	0.29589	0.49676	0.31039	0.29083
1 second	Horizontal	0.28239	0.36617	0.28749	0.32647	0.21725	0.30561
	Vertical	0.38994	0.48683	0.27857	0.51392	0.28634	0.28264
0.5 second	Horizontal	0.2782	0.44688	0.35239	0.37896	0.2116	0.33454
	Vertical	0.44074	0.51527	0.28938	0.57836	0.26966	0.2803
Period	Axis	Archer 7	Archer 8	Archer 9	Archer 10	Archer 11	Archer 12
1.5 second	Horizontal	0.51476	0.61658	0.32616	0.31375	0.46336	1.0098
	Vertical	0.40427	0.52678	0.31849	0.31845	0.55677	0.78791
1 second	Horizontal	0.64274	0.69487	0.36856	0.30354	0.47959	1.1003
	Vertical	0.44655	0.56229	0.3416	0.30778	0.55872	0.72486
0.5 second	Horizontal	1.0553	0.78052	0.41292	0.27078	0.51932	1.1449
	Vertical	0.5876	0.65167	0.41594	0.33648	0.61209	0.74638

Table 2. The fitting errors of the proposed AR2 model with the offset

Period	Direction	Archer 1	Archer 2	Archer 3	Archer 4	Archer 5	Archer 6
1.5 second	Horizontal	0.27138	0.33739	0.27805	0.35719	0.20572	0.3214
	Vertical	0.37356	0.44606	0.2957	0.49643	0.31024	0.29042
1 second	Horizontal	0.28238	0.36597	0.28123	0.32471	0.20626	0.30561
	Vertical	0.38743	0.48533	0.27854	0.51374	0.28221	0.28138
0.5 second	Horizontal	0.27814	0.44511	0.34122	0.37485	0.20683	0.33312
	Vertical	0.43877	0.51512	0.28822	0.57083	0.26964	0.27422
Period	Axis	Archer 7	Archer 8	Archer 9	Archer 10	Archer 11	Archer 12
1.5 second	Horizontal	0.51305	0.61635	0.32262	0.31371	0.46263	1.0097
	Vertical	0.40028	0.52651	0.31687	0.31844	0.55356	0.75776
1 second	Horizontal	0.64032	0.69485	0.36726	0.30325	0.47826	1.0998
	Vertical	0.44454	0.5612	0.3405	0.30748	0.553	0.70551
0.5 second	Horizontal	1.0359	0.78018	0.41264	0.2652	0.51721	1.1449
	Vertical	0.5791	0.65063	0.41231	0.33463	0.6077	0.72707

Table 3. The offset of the AR2 model

Period	Direction	Archer 1	Archer 2	Archer 3	Archer 4	Archer 5	Archer 6
1.5 second	Horizontal	-0.00058052	0.01033	-0.019272	-0.0038884	-0.025529	-0.023733
	Vertical	-0.041073	-0.014966	-0.0061132	-0.0097442	0.0036404	0.006234
1 second	Horizontal	0.0010416	0.0059878	-0.023603	-0.012531	-0.03227	-0.0018297
	Vertical	-0.037973	-0.018779	-0.0025786	-0.0075009	0.018714	0.010702
0.5 second	Horizontal	-0.0023238	0.018034	-0.032168	-0.019247	-0.024541	-0.030653
	Vertical	-0.033421	-0.0060354	-0.014988	-0.048478	0.0011309	0.026285
Period	Direction	Archer 7	Archer 8	Archer 9	Archer 10	Archer 11	Archer 12
1.5 second	Horizontal	-0.012295	0.0059099	0.017382	0.0050912	0.0099489	0.004111
	Vertical	-0.078546	-0.0094663	-0.044916	-0.0009664	0.054824	-0.098492
1 second	Horizontal	-0.014505	0.0019844	0.010435	0.014069	0.01313	0.013492
	Vertical	-0.059226	-0.01917	-0.038226	-0.0052068	0.067602	-0.11019
0.5 second	Horizontal	-0.040773	-0.0074621	0.0048441	0.062128	0.015876	-0.0016199
	Vertical	-0.1169	-0.019238	-0.074371	-0.012973	0.068057	-0.15667

Table 4. The ranking

Archer	1	2	3	4	5	6	7	8	9	10	11	12
Ranking	1	8	10	11	2	5	9	5	12	7	3	4

4. Conclusions

In this paper, the AR2 model has been proposed to establish the time sequence of the aiming trajectory. The estimation of the associated coefficients of the model is also formulated and implemented to obtain the estimated coefficients, and the corresponding fitting errors are also calculated based on three different time periods. The additional offset term can improve the fitting errors as expected, and most of archers whose better fitting errors are located at the 1.5 second period, but there are three archers and two archers fallen into the one second period and the half second period, respectively. This phenomenon is worth for further study. Moreover, the archers with good performance usually fallen into the extreme cases related to the offset term added in the AR2 model, this situation is also worth to find the causes.

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