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Fuzzy Adaptive Control of Nonlinear Stochastic Systems and its Application to Wireless Networks (I)

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***Abstract*— In this study, state estimation problem for the stochastic T-S fuzzy model with state-dependent noises on the system matrix and the output matrix is attacked. First, we derive sufficient conditions for a class of standard fuzzy state observers to ensure that the state estimation error is mean square bounded. The observer gain matrices in the fuzzy observer can be obtained by solving a linear matrix inequality (LMI). Then, the robust H_∞ fuzzy filtering problem is considered to minimize the worst-case ratio of the power of state estimation error to that of the external noises. The H_∞ observer gain matrices can be obtained by solving two linear matrix inequalities. To further improve estimation performance, we study the optimal Kalman fuzzy filtering problem with known statistical information of the process noise and the measurement noise. It is shown that the the minimum-variance estimation for the uncertain stochastic T-S fuzzy model is actually a linear estimation problem from the viewpoint of conditional expectation. The structure of the developed optimal Kalman fuzzy filter also very resembles that of the conventional Kalman filter. Comparison of estimation performances of the developed three estimators is made via simulation study.**

***Index Terms*— Stochastic T-S fuzzy, fuzzy H_∞ filter, fuzzy Kalman filter**

I. INTRODUCTION

For linear stochastic systems with parametric or non-parametric uncertainty, the robust state estimation problem has been well addressed. A robust Kalman filter for linear time-varying systems with stochastic parametric uncertainties has been constructed in [1]. In [2], a multi-objective robust state estimator for stochastic discrete time-delay systems with both deterministic and stochastic uncertainties is considered, where it is shown that the state estimation error is asymptotically stable provided some linear matrix inequalities (LMI) hold. In [3], a robust Kalman filter algorithm is presented for linear uncertain time-lag systems with randomly jumping parameters, and exponential stochastic stability for the state estimator is ensured. The above mentioned state estimators are designed based on the LMI technique [4] for linear stochastic systems.

State estimation of a nonlinear stochastic system is a more difficult and complex problem which attracts a lot of researchers' attention in recent years. Based on linearization technique, the extended Kalman filter has been well developed for state estimation of a nonlinear stochastic system [5]. State estimation based on feedback linearization using a recurrent neurofuzzy network with an analysis of variance decomposition structure can be referred to [6]. Recently, the Takagi and Sugeno (T-S) fuzzy model [7] has been used to deal with the state estimation problem for nonlinear systems. By interpolating the fuzzy IF-THEN rules representing the local linear models, the T-S fuzzy system can closely approximate the input-output relation of a nonlinear system. State estimation by designing a local fuzzy observer for each local linear model in the deterministic T-S fuzzy model by solving a set of LMI's can be traced back to [8]. Optimal output predictor for a stochastic T-S fuzzy ARMAX model by interpolating the local optimal predictor for each local ARMAX model has been proposed in [9]. In [10], the LMI technique is used to design an H_∞ robust fuzzy filter under the T-S fuzzy structure to minimize the worst-case effect of bounded disturbances and noise upon state estimation error. Without using the T-S fuzzy structure, optimal state estimation based on a fuzzy dynamic model is considered in [11] for nonlinear systems subject to non-Gaussian noise.

Although T-S fuzzy model has been used for state estimation for nonlinear systems, it seems that little attention has been paid to the state estimation problem for uncertain stochastic T-S fuzzy systems with state-dependent noises. Robust fuzzy filter design for continuous-time stochastic system with state-dependent noise in the system matrix is recently proposed in [12]. In this study, state estimation problem for descrete-time stochastic T-S fuzzy models with state dependent noises in the system matrix and the output function is addressed. First, based on the LMI technique, a class of standard state estimators will be attained so that the mean square state estimation error is bounded. Moreover, state estimation problem from the viewpoint of H_∞ filter theory will be attacked. The optimal H_∞ fuzzy filter will also be obtained by using the LMI method to minimize the maximal ratio of the power of state estimation error to that of the external noises. Finally, to further improve estimation performance, we shall study the optimal estimation problem for the T-S fuzzy model. Due to the state-dependent noise in the system, the coefficients in the T-S fuzzy model are randomly varying, and, from the control viewpoint, the

T-S fuzzy model is a nonlinear system. However, from the viewpoint of conditional expectation, we shall show that the state estimation problem for the stochastic T-S fuzzy model with state-dependent noise can be viewed as a linear state estimation problem. For state estimation of linear systems, the Kalman filter is an unbiased and minimum variance estimator under the Gaussian assumption. Based on the above observation, we shall derive the optimal fuzzy Kalman filter for the considered stochastic T-S fuzzy system under Gaussian assumption.

This study is organized as follows. Related materials of probability are introduced in Section II. In Section III, the considered stochastic T-S fuzzy model with state-dependent noise is described. In Section IV, a class of standard state estimators for the considered stochastic fuzzy systems is presented and analysis of mean square of estimation error is made. The optimal H_∞ fuzzy filter is discussed in Section V. The optimal fuzzy Kalman filter under Gaussian assumption is studied in Section VI. Simulation study is made in Section VII. Conclusions and discussions are given in Section VIII.

Notations

1) For a random vector x, its covariance matrix is denoted by Σ_x with $\Sigma_x = E\left\{(x-\hat{x})(x-\hat{x})^T\right\}$, where $\hat{x} = E\{x\}$ is the ensemble mean.
2) Let x, y, and z be three random vectors. The conditional expectation of x given y is denoted by $E\{x|y\}$. The conditional covariance of x given z is defined as
$$\Sigma_{x,x|z} = E\left\{(x - E\{x|z\})(x - E\{x|z\})^T|z\right\}$$
The conditional cross-covariance of x and y given z is defined as
$$\Sigma_{x,y|z} = E\left\{(x - E\{x|z\})(y - E\{y|z\})^T|z\right\}$$
Similarly, the conditional covariance of x given y and z is defined as
$$\Sigma_{x,x|y,\ z} = E\left\{(x - E\{x|y,z\})(x - E\{x|y,z\})^T|y,\ z\right\}$$
3) For a vector x and a matrix A, $\|x\|$ is the Euclidean norm of x and $\|A\|$ is the associated matrix induced norm. The minimal and maximal eigenvalues of A with only real eigenvalues are denoted as $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$, respectively. For a random vector x and a deterministic or random matrix A, the norm $\|A\|_{ms}$ is defined as $\|A\|_{ms}^2 \triangleq \sup_{E\{\|x\|^2\}=1} E\left\{\|Ax\|^2\right\}$. A stochastic process x_k is said to be mean square bounded if $\sup_k E\left\{\|x_k\|^2\right\} < \infty$.

II. Preliminary Materials Of Conditional Probability

In this section, we summarize some basic materials concerning conditional probability, which will be used in the derivation of the optimal fuzzy Kalman filter. Most of the materials presented in this section are extracted from [13].

Let $\mathcal{F}^\eta$ be the σ-algebra generated by the random variable η and let $E\{x\}$ exist. Let $\{\mathcal{F}_i\}_{i=1}^\infty$ be a set of nondecreasing σ-algebras. Some properties of conditional expectation are summarized in the following:

1) $E\{E\{x|\mathcal{F}_1\}\} = E\{x\}$.
2) If x is $\mathcal{F}_1-$measurable, $|x| < \infty$ a.s., and $E\{|y|\,|\mathcal{F}_1\} < \infty$ a.s., then $E\left\{x^T y|\mathcal{F}_1\right\} = x^T E\{y|\mathcal{F}_1\}$ a.s.
3) If $\mathcal{F}_1 \subset \mathcal{F}_2 \subset \mathcal{F}$, then $E\{E\{x|\mathcal{F}_2\}|\mathcal{F}_1\} = E\{x|\mathcal{F}_1\}$.

The concept of conditional independence is introduced below.

Definition 1: Random vectors x and y are called conditionally independent given z if
$$E\left\{e^{i\lambda^T x + i\mu^T y}|z\right\} = E\left\{e^{i\lambda^T x}|z\right\} E\left\{e^{i\mu^T y}|z\right\}$$
where $E\left\{e^{i\lambda^T x}|z\right\}$ and $E\left\{e^{i\mu^T y}|z\right\}$ are the conditional characteristic functions of x and y, respectively.

Lemma 1: (Theorem 1.10 in [13]) (i) If y and $\begin{bmatrix} x^T & z^T \end{bmatrix}^T$ are independent, then x and y are conditionally independent given z. (ii) If x and y are conditionally independent given z, then for any Borel set B,
$$P(\{x \in B\}|y,\ z) = P(\{x \in B\}|z)$$
and
$$E\{x|y,\ z\} = E\{x|z\}$$

Definition 2: Let $|x| < \infty$ a.s. Then x is called conditionally Gaussian given y if there exist an $\mathcal{F}^y$-measurable random vector $\hat{x}$ and an $\mathcal{F}^y$-measurable random matrix $\Sigma = \Sigma^T \geq 0$ a.s. such that the conditional characteristic function of x given y is expressed by
$$E\left\{e^{i\lambda^T x}|y\right\} = \exp\left(i\lambda^T\hat{x} - \frac{1}{2}\lambda^T\Sigma\lambda\right) \text{ a.s.} \quad (1)$$
for every constant vector λ. In this case
$$\hat{x} = E\{x|y\},\ \Sigma = E\left\{(x-\hat{x})(x-\hat{x})^T|y\right\} \text{ a.s.} \quad (2)$$

Note that if x is conditionally Gaussian given y and $A(\cdot)$ as well as $b(\cdot)$ are Borel measurable functions with almost surely $\|A(y)\| < \infty$ and $\|b(y)\| < \infty$, then it follows that $A(y)x + b(y)$ is also conditionally Gaussian given y.

Lemma 2: If random variable x is independent of random variables y and z, then
$$E\{xy|z\} = E\{x\}E\{y|z\}$$

Proof: Since x is independent of y and z, it follows
$$f_{xy|z}(x,\ y|z) = f_x(x)\frac{f_{yz}(y,\ z)}{f_z(z)} = f_x(x)f_{y|z}(y|z)$$
and thus
$$\begin{aligned} E\{xy|z\} &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy f_{xy|z}(x,\ y|z)\,dxdy \\ &= \int_{-\infty}^{\infty} x f_x(x)\,dx \int_{-\infty}^{\infty} y f_{y|z}(y|z)\,dy \\ &= E\{x\}E\{y|z\} \end{aligned}$$
■

Lemma 3: Suppose that the zero-mean random variables q_1, q_2, and the random vector v are mutually independent and are all independent of x and z. Also assume that $A(\cdot)$, $b(\cdot)$, $\Delta A(\cdot)$, $\Delta C(\cdot)$, and $\Delta b(\cdot)$ are Borel measurable functions with almost surely $\|A(\alpha)\| < \infty$, $\|b(\alpha)\| < \infty$, $\|\Delta A(\alpha)\| < \infty$, $\|\Delta C(\alpha)\| < \infty$, and $\|\Delta b(\alpha)\| < \infty$. If x is conditionally Gaussian given

z and α is $\mathcal{F}^z$-measurable, then $F(\alpha, x, q_1, q_2, v) = (A(\alpha) + \Delta A(\alpha)q_1 + \Delta C(\alpha)q_2 + \Delta C(\alpha)\Delta A(\alpha)q_1q_2)\, x + (b(\alpha) + \Delta b(\alpha)q_2 + B_0 v)$ is conditionally Gaussian given z, where B_0 is a constant matrix,

Proof: By Lemma 2, we have

$$\begin{aligned} & E\{F(\alpha, x, q_1, q_2, v)|z\} \\ = \;& A(\alpha) E\{x|z\} + \Delta A(\alpha) E\{q_1 x|z\} + \Delta C(\alpha) E\{q_2 x|z\} \\ & + \Delta C(\alpha)\Delta A(\alpha) E\{q_1 q_2 x|z\} + b(\alpha) \\ = \;& A(\alpha) E\{x|z\} + \Delta A(\alpha) E\{q_1\} E\{x|z\} \\ & + \Delta C(\alpha) E\{q_2\} E\{x|z\} \\ & + \Delta C(\alpha)\Delta A(\alpha) E\{q_1 q_2\} E\{x|z\} + b(\alpha) \\ = \;& A(\alpha) E\{x|z\} + b(\alpha) \end{aligned} \tag{3}$$

Hence $(A(\alpha) + \Delta A(\alpha)q_1 + \Delta C(\alpha)q_2 + \Delta C(\alpha)\Delta A(\alpha)q_1q_2)\, x + (b(\alpha) + \Delta b(\alpha)q_2 + B_0 v)$ is conditionally Gaussian given z. ■

Lemma 4: (Lemma 3.1 in [13]) If $w = \begin{bmatrix} x \\ y \end{bmatrix}$ is conditionally Gaussian given z, then x and y are conditionally independent given z if and only if

$$\Sigma_{xy|z} = E\left\{(x - E(x|z))(y - E(y|z))^T |z\right\} = 0 \text{ a.s.} \tag{4}$$

Lemma 5: (Lemma 3.2 in [13]) Let $\begin{bmatrix} x \\ y \end{bmatrix}$ be conditionally Gaussian given z with conditional covariance

$$\begin{bmatrix} \Sigma_{xx|z} & \Sigma_{xy|z} \\ \Sigma_{yx|z} & \Sigma_{yy|z} \end{bmatrix} \text{ a.s.}$$

Then (i) Given $(y,\ z)$, x is conditionally Gaussian with conditional mean

$$E\{x|z,\ y\} = E\{x|z\} + \Sigma_{xy|z}\Sigma_{yy|z}^{+}(y - E\{y|z\}) \tag{5}$$

and conditional covariance

$$\Sigma_{xx|z,\ y} = \Sigma_{xx|z} - \Sigma_{xy|z}\Sigma_{yy|z}^{+}\Sigma_{yx|z} \tag{6}$$

where $\Sigma_{yy|z}^{+}$ is the pseudo inverse of $\Sigma_{yy|z}$.

(ii) Given z, $x - E\{x|z,\ y\}$ is conditionally Gaussian and independent of y.

III. The Stochastic T-S Fuzzy Model

Consider the following stochastic T-S fuzzy model:

System Rule $i, 1 \le i \le L$:
IF $z_{1,k}$ is F_{i1} and $z_{2,k}$ is F_{i2} and ...... and $z_{g,k}$ is $F_{i,g}$,
THEN$x_{k+1} = (A_i + \Delta A_i(k))\, x_k + B_i u_k + w_k$
Measurement Rule $i, 1 \le i \le L$:
IF $z_{1,k-1}$ is F_{i1} and $z_{2,k-1}$ is F_{i2} and ...... and $z_{g,k-1}$ is $F_{i,g}$,
THEN $y_k = (C_i + \Delta C_i(k))x_k + v_k$ (7)

where L is the number of IF-THEN rules, g is the number of premise variables, F_{ij} is the fuzzy set for $1 \le i \le L$ and $1 \le j \le g$, and $z_{1,k},\ \ldots,\ z_{g,k}$ are the premise variables. Additionally, x_k is the state vector, y_k is the measurement output, u_k is the control input, and A_i, B_i, C_i are known constant matrices. The stochastic uncertainties in the system function and the output function are defined as $\Delta A_i(k) = \Gamma_{A_i} q_i(k)$ and $\Delta C_i(k) = \Gamma_{c_i} q_i(k)$, respectively. The driving noises $q_i(k)$ is an $i.i.d.$ (independent and identically distributed) processes of normal distribution $N(0,1)$. The zero-mean noises w_k and v_k are process noise and measurement noise, respectively. Then the overall stochastic T-S fuzzy system (7) is equivalent to

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^{L} h_i(z_k)\{(A_i + \Delta A_i(k))\, x_k + B_i u_k + w_k\} \\ y_k &= \sum_{i=1}^{L} h_i(z_{k-1})\{(C_i + \Delta C_i(k))x_k + v_k\} \end{aligned} \tag{8}$$

where $z_k = [z_{1,k}\ z_{2,k}\ \ldots\ z_{g,k}]$ is the vector of premise variables,

$$\begin{aligned} h_i(z_k) &= \frac{\mu_i(z_k)}{\sum_{i=1}^{L}\mu_i(z_k)} \\ \mu_i(z_k) &= \prod_{j=1}^{g} F_{ij}(z_{j,k}), \end{aligned} \tag{9}$$

and $F_{ij}(z_{j,k})$ is the grade of membership of $z_{j,k}$ in F_{ij}. It is assumed that $\sum_{i=1}^{L}\mu_i(z_k) > 0$ for any z_k. By the definition of $h_i(z_k)$, it follows

$$h_i(z_k) \ge 0\ ,\ \sum_{i=1}^{L} h_i(z_k) = 1 \tag{10}$$

The T-S fuzzy system in (8) can be rewritten into a more compact form as

$$\begin{aligned} x_{k+1} &= (A_k + \Delta A_k)\, x_k + B_k u_k + w_k \\ y_k &= (C_{k-1} + \Delta C_{k-1})x_k + v_k \end{aligned} \tag{11}$$

where

$$\begin{aligned} & A_k = \sum_{i=1}^{L} h_i(z_k) A_i,\ \Delta A_k = \sum_{i=1}^{L} h_i(z_k)\Delta A_i(k), \\ & C_{k-1} = \sum_{i=1}^{L} h_i(z_{k-1}) C_i,\ \Delta C_{k-1} = \sum_{i=1}^{L} h_i(z_{k-1})\Delta C_i(k), \\ & \Delta A_i(k) = \Gamma_{A_i} q_i(k),\ \Delta C_i(k) = \Gamma_{c_i} q_i(k), \\ & B_k = \sum_{i=1}^{L} h_i(z_k) B_i. \end{aligned} \tag{12}$$

We shall also make the following definitions:

$$\Gamma_{A_k} = \sum_{i=1}^{L} h_i(z_k)\Gamma_{A_i}, \Gamma_{C_k} = \sum_{i=1}^{L} h_i(z_{k-1})\Gamma_{C_i} \tag{13}$$

Let $\mathcal{F}_k$ be the $\sigma-$algebra generated by $Y_k \triangleq \{y_0,\ \ldots, y_k\}$. Some assumptions with respect to the T-S fuzzy model in (11) are made in the following.

(A1) $E\{q_i(k)q_j(m)\} = \sigma^2\delta(i-j)\delta(k-m)$.

(A2) $E\{w_k w_j^T\} = R_w\delta(k-j)$, $E\{v_k v_j^T\} = R_v\delta(k-j)$, $E\{q_i(k)|\mathcal{F}_{k-1}\} = 0,\ and\ E\left\{q_i^2(k)|\mathcal{F}_{k-1}\right\} = \sigma^2$.

(A3) The premise variables $z_{1,k}$ ...... $z_{g,k}$ are $\mathcal{F}_k-$measurable. The input u_k is dependent on Y_k and thus is also $\mathcal{F}_k-$measurable.

(A4) $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$ is independent of $\{w_k\ , v_k\}_{k\ge 0}$ and x_0 is conditionally Gaussian given y_0 with conditional mean $\hat{x}_0$ and conditional covariance Σ_0.

From assumption **(A1)**, it follows that

$$\begin{aligned} E\{\Delta A_i^T(k)\Delta A_j(m)\} &= \sigma^2\Gamma_{A_i}^T\Gamma_{A_j}\delta(i-j)\delta(k-m) \\ E\{\Delta C_i^T(k)\Delta C_j(k)\} &= \sigma^2\Gamma_{C_i}^T\Gamma_{C_j}\delta(i-j). \end{aligned}$$

IV. A Class Of Standard State Estimators for Stochastic Fuzzy Systems

With the T-S fuzzy system in (7), a class of standard state estimators, which are extensively used in the literature such as [8], [14], and [11], is given by

$$\begin{aligned} \hat{x}_{k+1} &= \sum_{i=1}^{L} h_i\left(z_k\right) A_i \hat{x}_k + \sum_{i=1}^{L} h_i\left(z_k\right) B_i u_k \\ &+ \sum_{i=1}^{L} h_i\left(z_k\right) L_i \left[y_k - \hat{y}_k\right] \\ \hat{y}_k &= \sum_{i=1}^{L} h_i\left(z_{k-1}\right) C_i \hat{x}_k \end{aligned} \tag{14}$$

where the filter gain L_i is related to the local linear model in the i-th rule. With the fuzzy system in (11) and the state estimator in (14), the state estimation error $\tilde{x}_{k+1} = x_{k+1} - \hat{x}_{k+1}$ can be expressed as

$$\begin{aligned} &\tilde{x}_{k+1} \\ = &\sum_{i=1}^{L}\sum_{j=1}^{L} h_i\left(z_k\right) h_j\left(z_{k-1}\right)\left(A_i - L_i C_j\right)\tilde{x}_k \\ &+\sum_{i=1}^{L}\sum_{j=1}^{L} h_i\left(z_k\right) h_j\left(z_{k-1}\right)\left(\Delta A_i\left(k\right) - L_i \Delta C_j(k)\right) x_k \\ &+\sum_{i=1}^{L} h_i\left(z_k\right)\left(w_k - L_i v_k\right) \end{aligned} \tag{15}$$

With the fuzzy system in (8) and the state estimation error in (15), the overall state-space model with the augmented state vector $\bar{x}_k = \left[\begin{array}{cc} x_{k+1}^T & \tilde{x}_{k+1}^T \end{array}\right]^T$ can be expressed as

$$\bar{x}_{k+1} = \tilde{F}_{ij}(k)\bar{x}_k + \eta_{k+1} \tag{16}$$

where

$$\begin{aligned} \tilde{F}_{ij}(k) &= \sum_{i=1}^{L}\sum_{j=1}^{L} h_i\left(z_k\right) h_j\left(z_{k-1}\right)\left(F_{ij}\bar{x}_k + \Delta F_{ij}(k)\bar{x}_k\right), \\ \eta_{k+1} &= \sum_{i=1}^{L} h_i\left(z_k\right)\left(\tilde{B}_i u_k + \tilde{E}_i \tilde{w}_k\right), \\ F_{ij} &= \left[\begin{array}{cc} A_i & 0 \\ 0 & A_i - L_i C_j \end{array}\right], \\ \Delta F_{ij}(k) &= \left[\begin{array}{cc} \Delta A_i(k) & 0 \\ \Delta A_i(k) - L_i \Delta C_j(k) & 0 \end{array}\right], \\ \tilde{B}_i &= \left[\begin{array}{c} B_i \\ 0 \end{array}\right],\ \tilde{E}_i = \left[\begin{array}{cc} 0 & I \\ -L_i & I \end{array}\right], \\ \tilde{w}_k &= \left[\begin{array}{c} v_k \\ w_k \end{array}\right]. \end{aligned}$$

By assumption **(A2)** imposed on the driving noise w_k and v_k, it follows that

$$\sup_k E\left(\left\|\eta_{k+1}\right\|^2\right) \le \bar{\sigma}_\eta^2 < \infty$$

Before deriving the mean-square stability result for the forced stochastic fuzzy dynamic equation (16), we shall first consider the following unforced stochastic fuzzy system

$$\bar{x}_{k+1} = \tilde{F}_{ij}(k)\bar{x}_k \tag{17}$$

A sufficient condition concerning the mean-square exponential stability of the unforced system (17) is given in the following theorem.

Lemma 6: If there exists a symmetric positive definite matrix $P = \left[\begin{array}{cc} P_1 & 0 \\ 0 & P_2 \end{array}\right]$ such that the linear matrix inequalities

$$\left[\begin{array}{cccccccc} \lambda_1 P_1 & 0 & A_i^T P_1 & 0 & \Gamma_{A_i}^T P_1 & \Gamma_{A_i}^T P_2 & 0 & -\Gamma_{c_j}^T K_i^T \\ 0 & \lambda_2 P_2 & 0 & A_i^T P_2 - C_j^T K_i^T & 0 & 0 & 0 & 0 \\ P_1 A_i & 0 & P_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & P_2 A_i - K_i C_j & 0 & P_2 & 0 & 0 & 0 & 0 \\ P_1 \Gamma_{A_i} & 0 & 0 & 0 & \frac{P_1}{2\sigma^2} & 0 & 0 & 0 \\ P_2 \Gamma_{A_i} & 0 & 0 & 0 & 0 & \frac{P_2}{2\sigma^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{P_1}{2\sigma^2} & 0 \\ -K_i \Gamma_{c_j} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{P_2}{2\sigma^2} \end{array}\right] > 0 \tag{18}$$

hold for $1 \le i \le L$ and $1 \le j \le L$ where $K_i = P_2 L_i$, then

$$\begin{aligned} &\Lambda P - \sum_{i=1}^{L}\sum_{j=1}^{L} h_i\left(z_k\right) h_j\left(z_{k-1}\right) \\ &\left(F_{ij}^T P F_{ij} + 2\sigma^2\left(\Omega_1^T P \Omega_1 + \Omega_2^T P \Omega_2\right)\right) > 0 \end{aligned} \tag{19}$$

where

$$\Omega_1 = \left[\begin{array}{cc} \Gamma_{A_i} & 0 \\ \Gamma_{A_i} & 0 \end{array}\right] \quad \Omega_2 = \left[\begin{array}{cc} 0 & 0 \\ -L_i \Gamma_{C_j} & 0 \end{array}\right]$$
$$\Lambda = \left[\begin{array}{cc} \lambda_1 I & 0 \\ 0 & \lambda_2 I \end{array}\right]$$

Proof: Note that by the definition of the function $h_i\left(z_k\right)$, to ensure (19), it suffices to guarantee

$$\Lambda P - \left(F_{ij}^T P F_{ij} + 2\sigma^2\left(\Omega_1^T P \Omega_1 + \Omega_2^T P \Omega_2\right)\right) > 0 \tag{20}$$

By Schur complement, (20) is equivalent to (18). This completes the proof. ■

Theorem 1: If there exist symmetric positive definite matrices P_1 and P_2 such that the matrix inequalities (18) hold for λ_1 and λ_2 with $0 \le \lambda_1,\ \lambda_2 < 1$, then the stochastic fuzzy system (17) is mean square exponentially stable with

$$E\left\{\left\|\bar{x}_k\right\|^2\right\} \le \frac{\lambda_p^{\max}}{\lambda_p^{\min}} \lambda^{k-k_0} E\left\{\left\|\bar{x}_{k_0}\right\|^2\right\},\ \forall k \ge k_0 \tag{21}$$

where k_0 is an arbitrary initial time, $\bar{x}_{k_0}$ is an arbitrary initial condition, the positive constants λ, $\lambda_p^{\min}$ and $\lambda_p^{\max}$ are defined as $\lambda = \max(\lambda_1, \lambda_2)$, $\lambda_p^{\min} = \min_{1\le i\le 2}\left(\lambda_{\min}\left(P_i\right)\right)$ and $\lambda_p^{\max} = \max_{1\le i\le 2}\left(\lambda_{\max}\left(P_i\right)\right)$, respectively.

Proof: First define a Lyapunov function as

$$\begin{aligned} V\left(\bar{x}_k\right) &= x_k^T P_1 x_k + \tilde{x}_k^T P_2 \tilde{x}_k \\ &= \bar{x}_k^T P \bar{x}_k \end{aligned} \tag{22}$$

where the matrices P_i , $1 \le i \le 2$, are positive definite matrices satisfying (18). Then we have

$$\lambda_p^{\min} E\{||\bar{x}_k||^2\} \le V(\bar{x}_k) \le \lambda_p^{\max} E\{||\bar{x}_k||^2\} \qquad (23)$$

By referring to (17), the unforced state-space fuzzy system model is

$$\bar{x}_{k+1} = \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) (F_{ij} + \Delta F_{ij}(k)) \bar{x}_k \qquad (24)$$

where

$$\begin{aligned} \Delta F_{ij}(k) &= \Omega_1 q_i(k) - \Omega_2 q_j(k) \\ \Omega_1 &= \begin{bmatrix} \Gamma_{A_i} & 0 \\ \Gamma_{A_i} & 0 \end{bmatrix} \\ \Omega_2 &= \begin{bmatrix} 0 & 0 \\ -L_i\Gamma_{C_j} & 0 \end{bmatrix} \end{aligned} \qquad (25)$$

It follows from the unforced system in (17) that

$$\begin{aligned} & V(\bar{x}_{k+1}) = \bar{x}_{k+1}^T P \bar{x}_{k+1} \\ = & (\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) (F_{ij} + \Delta F_{ij})\bar{x}_k)^T P \times \\ & (\sum_{m=1}^{L}\sum_{n=1}^{L} h_m(z_k) h_n(z_{k-1}) (F_{mn} + \Delta F_{mn})\bar{x}_k) \\ \le & \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) \bar{x}_k^T [F_{ij} + \Delta F_{ij}(k)]^T P \times \\ & [F_{ij} + \Delta F_{ij}(k)]\bar{x}_k \end{aligned} \qquad (26)$$

Let F'_k be the $\sigma-$algebra spanned by $\{y_s\}_{s \le k} \cup \{x_s\}_{s \le k}$. Now applying the conditional mean operator $E\{\cdot|F'_k\}$ to (26), we have

$$\begin{aligned} & E\{V(\bar{x}_{k+1})|F'_k\} \\ \le & E\left\{\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) \bar{x}_k^T \right. \\ & \left. (F_{ij}^T P F_{ij} + E\{\Delta F_{ij}^T(k) P \Delta F_{ij}(k)|F'_k\})\bar{x}_k\right\} \end{aligned} \qquad (27)$$

Note that $E\{q_i^T(k) q_j(k)|F'_k\} = E\{q_i^T(k) q_j(k)|F_k\}$ as $q_i(k)$ is independent of $\bar{x}_k$. With the definition of $\Delta F_{ij}(k)$ in (25), we have

$$\begin{aligned} & \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) E\{\Delta F_{ij}^T(k) P \Delta F_{ij}(k)|F'_k\} \\ = & \sigma^2\Omega_1^T P\Omega_1 + \sigma^2\Omega_2^T P\Omega_2 \\ & - \sum_{i=1}^{L} h_i^2(z_k)\left[\sigma^2(\Omega_2^T P\Omega_1) + \sigma^2\left(\Omega_1^T P\Omega_2\right)\right] \\ \le & \sigma^2\Omega_1^T P\Omega_1 + \sigma^2\Omega_2^T P\Omega_2 \\ & + \sum_{i=1}^{L} h_i(z_k)\left[\sigma^2\Omega_2^T P\Omega_2 + \sigma^2\Omega_1^T P\Omega_1\right] \\ \le & 2\sigma^2\left(\Omega_1^T P\Omega_1 + \Omega_2^T P\Omega_2\right) \end{aligned} \qquad (28)$$

Substituting (28) into (27), we can obtain

$$\begin{aligned} & E\{V(\bar{x}_{k+1})|F'_k\} \\ \le & \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) \bar{x}_k^T F_{ij}^T P F_{ij}\bar{x}_k \\ & + 2\sigma^2\bar{x}_k^T\left(\Omega_1^T P\Omega_1 + \Omega_2^T P\Omega_2\right)\bar{x}_k \\ = & \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) \bar{x}_k^T [F_{ij}^T P F_{ij} \\ & + 2\sigma^2\left(\Omega_1^T P\Omega_1 + \Omega_2^T P\Omega_2\right)]\bar{x}_k \\ \le & \lambda\bar{x}_k^T P\bar{x}_k \\ = & \lambda V(\bar{x}_k) \end{aligned} \qquad (29)$$

where the result in Lemma 6 is used in the above derivation and $\lambda = \max(\lambda_1,\ \lambda_2)$. Applying the conditional expectation operator $E\{\cdot\}$ again to the both sides of (29), we have

$$E\{V(\bar{x}_{k+1})\} \le \lambda E\{V(\bar{x}_k)\}$$

which implies

$$E\{V(\bar{x}_k)\} \le \lambda^{k-k_0} E\{V(\bar{x}_{k_0})\}$$

Finally, using the fact of (23), inequality (21) is obtained. ■

For the forced system in (16), with the exponential stability of the related unforced system and the uniform mean-square bounded property of the forced term η_{k+1}, we readily have the following result by referring to Theorem 2 in [9].

Theorem 2: Assume that the initial state x_0 and initial estimation error $\tilde{x}_0$ of the system (16) are mean-square bounded, i.e., $E\{\bar{x}_0^2\} < \infty$. If there exist symmetric positive definite matrices P_1 and P_2, such that the matrix inequalities (18) hold for λ_1 and λ_2 with $0 \le \lambda_1,\ \lambda_2 < 1$, then $\bar{x}_k$ is mean square bounded.

V. Optimal H_∞ Filter For Stochastic Fuzzy Systems

In this section, we shall discuss the H_∞ filter design problem for the stochastic fuzzy system in (8) and the state estimator structure in (14). From (16), the augmented system including the system and the estimation error dynamics can be expressed as

$$\bar{x}_{k+1} = \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) (\tilde{F}_{ij}(k)\bar{x}_k + \tilde{E}_i\tilde{w}_k) \qquad (30)$$

Let us consider the following suboptimal H_∞ performance for the augmented system

$$E\left\{\sum_{k=1}^{N} \bar{x}_{k+1}^T Q\bar{x}_{k+1}\right\} \le E\left\{\bar{x}_0^T P\bar{x}_0 + \rho^2\sum_{k=1}^{N}\tilde{w}_k^T\tilde{w}_k\right\} \qquad (31)$$

where ρ is a prescribed noise attenuation level, and P is a positive definite weighting matrix. Here we set $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$ and $Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}$.

Theorem 3: If there exist matrices K_i for $1 \leq i \leq L$ and positive definite matrices P_1 as well as P_2 such that the following linear matrix inequalities

$$\begin{bmatrix} \frac{P_1-Q_1}{2} & 0 & A_i^T P_1 & 0 & \Gamma_{A_i}^T P_1 & \Gamma_{A_i}^T P_2 & 0 & -\Gamma_{c_j}^T K_i^T \\ 0 & \frac{P_2-Q_2}{2} & 0 & A_i^T P_2 - C_j^T K_i^T & 0 & 0 & 0 & 0 \\ P_1 A_i & 0 & P_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & P_2 A_i - K_i C_j & 0 & P_2 & 0 & 0 & 0 & 0 \\ P_1 \Gamma_{A_i} & 0 & 0 & 0 & \frac{P_1}{2\sigma^2} & 0 & 0 & 0 \\ P_2 \Gamma_{A_i} & 0 & 0 & 0 & 0 & \frac{P_2}{2\sigma^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{P_1}{2\sigma^2} & 0 \\ -K_i \Gamma_{c_j} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{P_2}{2\sigma^2} \end{bmatrix} > 0 \qquad (32)$$

$$\begin{bmatrix} \rho^2 I & 0 & 0 & -K_i^T \\ 0 & \rho^2 I & P_1 & P_2 \\ 0 & P_1 & \frac{P_1}{2} & 0 \\ -K_i & P_2 & 0 & \frac{P_2}{2} \end{bmatrix} > 0 \qquad (33)$$

hold for $1 \leq i \leq L$ and $1 \leq j \leq L$, then the suboptimal H_∞ control performance in (31) is attained and the observer gain matrix L_i is obtained via the equation $K_i = P_2 L_i$, i.e., $L_i = P_2^{-1} K_i$.

Proof: Let us choose a Lyapunov function for the system (30) as

$$V(\bar{x}_k) = E\{\bar{x}_k^T P \bar{x}_k\} \qquad (34)$$

where P is a positive definite matrix. By the Lyapunov function, we obtain

$$\begin{aligned} & V(\bar{x}_{k+1}) - V(\bar{x}_k) \\ = \; & E\left\{ \left[\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) (\tilde{F}_{ij}(k)\bar{x}_k + \tilde{E}_i \tilde{w}_k) \right]^T P \right. \\ & \times \left[(\sum_{m=1}^{L}\sum_{n=1}^{L} h_m(z_k) h_n(z_{k-1}) (\tilde{F}_{mn}(k)\bar{x}_k + \tilde{E}_m \tilde{w}_k) \right] \\ & \left. -\bar{x}_k^T P \bar{x}_k \right\} \\ \leq \; & E\left\{ \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) \left[(\tilde{F}_{ij}(k)\bar{x}_k + \tilde{E}_i \tilde{w}_k)^T P \right. \right. \\ & \left.\left. (\tilde{F}_{ij}(k)\bar{x}_k + \tilde{E}_i \tilde{w}_k) \right] - \bar{x}_k^T P \bar{x}_k \right\} \\ \leq \; & 2E\left\{ \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) \bar{x}_k^T \tilde{F}_{ij}^T(k) P \tilde{F}_{ij}(k) \bar{x}_k \right\} \\ & +2E\left\{ \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) \tilde{w}_k^T \tilde{E}_i^T P \tilde{E}_i \tilde{w}_k \right\} \\ & -E\left\{ \bar{x}_k^T P \bar{x}_k \right\} \end{aligned} \qquad (35)$$

We shall separately analyze the first term and the second term at the right hand side of (35). First let $\mathcal{F}'_k$ be the $\sigma-$algebra spanned by $\{y_s\}_{s \leq k} \cup \{x_s\}_{s \leq k}$. Using the conditional mean operator $E\{\cdot|\mathcal{F}'_k\}$ upon the first term at the right hand side of (35), we have

$$\begin{aligned} & E\left\{ \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) \bar{x}_k^T \tilde{F}_{ij}^T(k) P \tilde{F}_{ij}(k) \bar{x}_k \right\} \\ = \; & E\left\{ E\left\{ \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) \bar{x}_k^T [F_{ij} + \Delta F_{ij}(k)]^T \right.\right. \\ & \left.\left. \times P[F_{ij} + \Delta F_{ij}(k)] \bar{x}_k | \mathcal{F}'_k \right\}\right\} \\ = \; & E\left\{ \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) \bar{x}_k^T \right. \\ & \left. E\left\{ [F_{ij} + \Delta F_{ij}(k)]^T P [F_{ij} + \Delta F_{ij}(k)] | \mathcal{F}'_k \right\} \bar{x}_k \right\} \\ = \; & E\left\{ \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) \right. \\ & \left. \bar{x}_k^T (F_{ij}^T P F_{ij} + E\left\{ \Delta F_{ij}^T(k) P \Delta F_{ij}(k) | \mathcal{F}'_k \right\}) \bar{x}_k \right\} \\ = \; & E\left\{ \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) \bar{x}_k^T F_{ij}^T P F_{ij} \bar{x}_k \right\} \\ & +E\left\{ \left[\bar{x}_k^T \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) \right.\right. \\ & \left.\left. E\left\{ \Delta F_{ij}^T(k) P \Delta F_{ij}(k) | \mathcal{F}'_k \right\} \bar{x}_k \right] \right\} \end{aligned} \qquad (36)$$

With the definition of $\Delta F_{ij}(k)$ in (25), assumptions (**A1**) and (**A2**), we get

$$\begin{aligned} & \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k) h_j(z_{k-1}) E\left\{ \Delta F_{ij}^T(k) P \Delta F_{ij}(k) | \mathcal{F}'_k \right\} \\ = \; & \sigma^2 \Omega_1^T P \Omega_1 + \sigma^2 \Omega_2^T P \Omega_2 \\ & - \sum_{i=1}^{L} h_i(z_k) h_i(z_{k-1}) \left[\sigma^2 (\Omega_2^T P \Omega_1) + \sigma^2 \left(\Omega_1^T P \Omega_2 \right) \right] \\ \leq \; & \sigma^2 \Omega_1^T P \Omega_1 + \sigma^2 \Omega_2^T P \Omega_2 \\ & + \sum_{i=1}^{L} h_i(z_k) h_i(z_{k-1}) \left[\sigma^2 \Omega_2^T P \Omega_2 + \sigma^2 \Omega_1^T P \Omega_1 \right] \\ \leq \; & 2\sigma^2 \left(\Omega_1^T P \Omega_1 + \Omega_2^T P \Omega_2 \right) \end{aligned} \qquad (37)$$

where we have used the fact that $E\left\{ q_i^T(k) q_j(k) | \mathcal{F}'_k \right\} = E\left\{ q_i^T(k) q_j(k) | \mathcal{F}_k \right\}$ as $q_i(k)$ is independent of $\bar{x}_k$. Substituting

(37) into (36), one can obtain

$$
\begin{aligned}
& E\left\{\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k)\, h_j(z_{k-1})\, \bar{x}_k^T \tilde{F}_{ij}^T(k) P \tilde{F}_{ij}(k)\bar{x}_k\right\} \\
\leq\ & E\left\{\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k)\, h_j(z_{k-1})\, \bar{x}_k^T F_{ij}^T P F_{ij}\bar{x}_k\right\} \\
& +2\sigma^2 E\left\{\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k)\, h_j(z_{k-1})\, \bar{x}_k^T \left(\Omega_1^T P \Omega_1\right.\right. \\
& \left.\left. +\Omega_2^T P\Omega_2\right)\bar{x}_k\right\} \\
=\ & E\left\{\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k)\, h_j(z_{k-1})\, \bar{x}_k^T \left[F_{ij}^T P F_{ij}\right.\right. \\
& \left.\left. +2\sigma^2\left(\Omega_1^T P\Omega_1+\Omega_2^T P\Omega_2\right)\right]\bar{x}_k\right\}
\end{aligned} \tag{38}
$$

Then, substituting (38) into (35), we have

$$
\begin{aligned}
& V(\bar{x}_{k+1})-V(\bar{x}_k) \\
\leq\ & E\left\{\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k)\, h_j(z_{k-1}) \times\right. \\
& \left.\left\{\bar{x}_k^T\left[2(F_{ij}^T P F_{ij}+2\sigma^2\left(\Omega_1^T P\Omega_1+\Omega_2^T P\Omega_2\right))-P\right]\bar{x}_k\right\}\right\} \\
& +E\left\{\sum_{i=1}^{L}\sum_{j=1}^{L} 2h_i(z_k)\, h_j(z_{k-1})\, \tilde{w}_k^T \tilde{E}_i^T P\tilde{E}_i\tilde{w}_k\right\}
\end{aligned} \tag{39}
$$

If there exist a positive matrix Q and a scalar ρ such that

$$
2(F_{ij}^T P F_{ij}+2\sigma^2\left(\Omega_1^T P\Omega_1+\Omega_2^T P\Omega_2\right))-P \quad < \quad -Q \tag{40}
$$

$$
2\tilde{E}_i^T P\tilde{E}_i \quad < \quad \rho^2 I \tag{41}
$$

then

$$
\begin{aligned}
& V(\bar{x}_{k+1})-V(\bar{x}_k) \\
\leq\ & E\left\{\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z_k)\, h_j(z_{k-1})\left(-\bar{x}_k^T Q_1\bar{x}_k\right)+\rho^2\tilde{w}_k^T\tilde{w}_k\right\} \\
\leq\ & E\left\{-\bar{x}_k^T Q_1\bar{x}_k+\rho^2\tilde{w}_k^T\tilde{w}_k\right\}
\end{aligned} \tag{42}
$$

Summing (42) from $k=0$ to $k=N$, we have

$$
V(\bar{x}_{N+1})-V(\bar{x}_0)\leq -E\left\{\sum_{k=1}^{N}\bar{x}_{k+1}^T Q_1\bar{x}_{k+1}+\rho^2\sum_{k=1}^{N}\tilde{w}_k^T\tilde{w}_k\right\} \tag{43}
$$

and by the definition of the Lyapunov function $V(\bar{x}_k)$ in (34), we get

$$
E\left\{\sum_{k=1}^{N}\bar{x}_{k+1}^T Q\bar{x}_{k+1}\right\}\leq E\left\{\bar{x}_0^T P\bar{x}_0+\rho^2\sum_{k=1}^{N}\tilde{w}_k^T\tilde{w}_k\right\} \tag{44}
$$

Therefore, the H_∞ control performance is achieved with a prescribed ρ^2 provided that the two inequalities (40) and (41) hold. By the Schur complement, (40) is equivalent to (32) by using $K_i = P_2 L_i$. Similarly, (41) is equivalent to (33). This completes the proof. ■

VI. Optimal Fuzzy Kalman Filter Under Gaussian Assumption

Due to the random coefficients in the stochastic nonlinear system (11), taking the conditional expectation $E\{\cdot|Y_k\}$ to both sides of (11) leads to

$$
\begin{aligned}
E\{x_{k+1}|Y_k\} &= A_k E\{x_k|Y_k\}+B_k u_k \text{ a.s.} \\
E\{y_{k+1}|Y_k\} &= C_k E\{x_{k+1}|Y_k\} \text{ a.s.}
\end{aligned}
$$

where the random terms u_k, A_k, B_k and C_k are available given the measurement data set Y_k. We can find that from the conditional expectation point of view, the state estimation problem of the considered stochastic T-S fuzzy model can be viewed as the state estimation problem of time-varying linear systems.

Lemma 7: For the stochastic system (11) under assumptions **(A1)-(A4)**, both x_k and $\begin{bmatrix} x_{k+1} \\ y_{k+1}\end{bmatrix}$ are conditionally Gaussian given $Y_k = \{y_0,\ y_1,\cdots,\ y_k\}$.

Proof: By assumption **(A4)** and the smoothing property of the conditional mean, we get

$$
\begin{aligned}
& E\left\{e^{i\lambda^T x_0+i\mu^T w_0}|y_0\right\} \\
=\ & E\left\{\left(e^{i\lambda^T x_0}\right)E\left\{e^{i\mu^T w_0}|x_0,\, y_0\right\}|y_0\right\} \\
=\ & E\left\{e^{i\mu^T w_0}\right\}E\left\{e^{i\lambda^T x_0}|y_0\right\} \\
=\ & \exp\left(i\lambda^T\hat{x}_0-\frac{1}{2}\lambda^T\Sigma_0\lambda-\frac{1}{2}\mu^T R_w\mu\right)
\end{aligned}
$$

It follows that $\begin{bmatrix} x_0 \\ w_0\end{bmatrix}$ is conditionally Gaussian given y_0 with finite conditional mean and covariance. Then, from the stochastic system equation (11), we can find that

$$
\begin{aligned}
& \begin{bmatrix} x_1 \\ y_1\end{bmatrix} \\
=\ & \left(\begin{bmatrix} A_0 & I \\ C_0A_0 & C_0\end{bmatrix}+\begin{bmatrix}\Delta A_0 & 0 \\ C_0\Delta A_0 & 0\end{bmatrix}\right)\begin{bmatrix} x_0 \\ w_0\end{bmatrix} \\
& +\begin{bmatrix} 0 & 0 \\ \Delta C_0 A_0 & \Delta C_0\end{bmatrix}+\begin{bmatrix} 0 & 0 \\ \Delta C_0\Delta A_0 & 0\end{bmatrix}\begin{bmatrix} x_0 \\ w_0\end{bmatrix} \\
& +\left(\begin{bmatrix} B_0u_0 \\ C_0B_0u_0\end{bmatrix}+\begin{bmatrix} 0 \\ \Delta C_0B_0u_0\end{bmatrix}+\begin{bmatrix} 0 \\ I\end{bmatrix}v_1\right)
\end{aligned}
$$

Note that $\begin{bmatrix} A_0 & I \\ C_0A_0 & C_0\end{bmatrix}$ and $\begin{bmatrix} B_0u_0 \\ C_0B_0u_0\end{bmatrix}$ are $\mathcal{F}_0-$measurable. By the definitions in (12), we have $\Delta A_0 = \sum_{i=1}^{L} h_i(z_0)\,\Gamma_{A_i}q_i(0)$ and $\Delta C_0 = \sum_{i=1}^{L} h_i(z_0)\,\Gamma_{c_i}q_i(1)$ so that by Lemma 3, both $\begin{bmatrix} x_1 \\ y_1\end{bmatrix}$ and x_0 are conditionally Gaussian given y_0 and the conditional means and conditional covariances are finite a.s. By induction, assume that given Y_{k-1}, both x_{k-1} and $\begin{bmatrix} x_k \\ y_k\end{bmatrix}$ are conditionally Gaussian with a.s. finite conditional means and conditional covariances. By Lemma 5, given Y_k, x_k is conditionally Gaussian with a.s.

finite conditional mean and conditionally covariance. Also note that w_k is independent of $\{x_s,\ y_s\}_{s\leq k}$. Since the conditional characteristic function

$$\begin{aligned} & E\left\{e^{i\lambda^T x_k+i\mu^T w_k}|Y_k\right\} \\ = \ & E\left\{E\left\{e^{i\lambda^T x_k}e^{i\mu^T w_k}|x_k,\ Y_k\right\}|Y_k\right\} \\ = \ & E\left\{\left(e^{i\lambda^T x_k}\right)E\left(e^{i\mu^T w_k}|x_k,\ Y_k\right)|Y_k\right\} \\ = \ & E\left\{e^{i\mu^T w_k}\right\}E\left\{e^{i\lambda^T x_k}|Y_k\right\} \\ = \ & \exp\left(i\lambda^T\hat{x}_k-\frac{1}{2}\lambda^T\Sigma_k\lambda-\frac{1}{2}\mu^T R_w\mu\right) \end{aligned}$$

is Gaussian, $\begin{bmatrix} x_k \\ w_k \end{bmatrix}$ is conditionally Gaussian with a.s. finite conditional mean and conditional variance given Y_k. In particular, given Y_k, x_k is conditionally Gaussian with a.s. finite conditional mean and conditional covariance.

Finally, with the system equation (8), we have

$$\begin{aligned} & \begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} \\ = \ & \begin{bmatrix} A_k+\Delta A_k & I \\ C_kA_k+C_k\Delta A_k+\Delta C_kA_k+\Delta C_k\Delta A_k & C_k+\Delta C_k \end{bmatrix}\begin{bmatrix} x_k \\ w_k \end{bmatrix} \\ & +\begin{bmatrix} B_ku_k \\ C_kB_ku_k+\Delta C_kB_ku_k+v_{k+1} \end{bmatrix} \end{aligned}$$

Similarly, by Lemma 3, we can conclude that given Y_k, both x_k and $\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix}$ are conditionally Gaussian with a.s. finite conditional means and conditional covariance. By induction, the proof is complete. ■

With the above lemma, the optimal Kalman filter algorithm is constructed in the follow theorem.

Theorem 4: For the stochastic fuzzy system in (11) with assumptions **(A1)-(A4)**, the state x_k is conditionally Gaussian given $Y_k=[y_0,y_1,\cdots,y_k]$, with conditional mean $\hat{x}_k$ and conditional covariance $\Sigma_k=(x_k-E(x_k|Y_k))(x_k-E(x_k|Y_k))^T$ as follows

$$\hat{x}_{k+1}=A_k\hat{x}_k+B_ku_k+K_k(y_{k+1}-C_kA_k\hat{x}_k-C_kB_ku_k) \tag{45}$$

$$\Sigma_{k+1}=R_k-K_kC_kR_k \tag{46}$$

with initial values $\hat{x}_0$ and Σ_0, where the time-varying filter gain K_k is given by

$$K_k=R_kC_k^T(C_kR_kC_k^T+R_v+\Lambda_k)^{-1} \tag{47}$$

and

$$R_k = A_k\Sigma_kA_k^T+\Psi_k+R_w \tag{48}$$

$$\Psi_k = \sigma^2\sum_{i=1}^{L}h_i^2(z_k)\Gamma_{A_i}(\Sigma_k+\hat{x}_k\hat{x}_k^T)\Gamma_{A_i}^T \tag{49}$$

$$\Lambda_k = \sigma^2\sum_{i=1}^{L}h_i^2(z_k)\Gamma_{C_i}\Xi_k\Gamma_{C_i}^T \tag{50}$$

$$\Xi_k = R_k+(A_k\hat{x}_k+B_ku_k)(A_k\hat{x}_k+B_ku_k)^T \tag{51}$$

The related one-step ahead prediction $\hat{x}_{k+1|k}\triangleq E\{x_{k+1}|Y_k\}$ is

$$\hat{x}_{k+1|k}=A_k\hat{x}_k+B_ku_k \tag{52}$$

and the prediction error covariance matrix $E\{(x_{k+1}-E\{x_{k+1}|Y_k\})(x_{k+1}-E\{x_{k+1}|Y_k\})^T|Y_k\}$ is equal to R_k.

Proof: Using Lemma 5, by identifying z with Y_k, y with y_{k+1}, and x with x_{k+1} in Lemma 7, we obtain

$$\begin{aligned} & E\{x_{k+1}|Y_k,\ y_{k+1}\} \\ = \ & E\{x_{k+1}|Y_{k+1}\} \\ = \ & E\{x_{k+1}|Y_k\}+\Sigma_{xy|z}\Sigma_{yy|z}^{+}(y_{k+1}-E\{y_{k+1}|Y_k\}) \text{ a.s.} \end{aligned} \tag{53}$$

where

$$\begin{aligned} \Sigma_{xy|z} = \ & E\{(x_{k+1}-E(x_{k+1}|Y_k)) \\ & \times(y_{k+1}-E(y_{k+1}|Y_k))^T|Y_k\} \end{aligned} \tag{54}$$

$$\begin{aligned} \Sigma_{yy|z} = \ & E\{(y_{k+1}-E(y_{k+1}|Y_k)) \\ & \times(y_{k+1}-E(y_{k+1}|Y_k))^T|Y_k\} \end{aligned} \tag{55}$$

Note that from the stochastic fuzzy system in (11), we have

$$E\{x_{k+1}|Y_k\} = A_kE\{x_k|Y_k\}+B_ku_k \text{ a.s.} \tag{56}$$

$$E\{y_{k+1}|Y_k\} = C_kE\{x_{k+1}|Y_k\} \text{ a.s.} \tag{57}$$

Before we compute $\Sigma_{xy|z}$ and $\Sigma_{yy|z}$, it is helpful to derive an expression for $R_k=\Sigma_{xx|z}$. By the stochastic fuzzy system in (11), we have

$$\begin{aligned} & R_k \\ = \ & E\left\{(x_{k+1}-E\{x_{k+1}|Y_k\})(x_{k+1}-E\{x_{k+1}|Y_k\})^T|Y_k\right\} \\ = \ & E\{[A_k(x_k-E(x_k|Y_k))+\Delta A_kx_k+w_k] \\ & \times[A_k(x_k-E(x_k|Y_k))+\Delta A_kx_k+w_k]^T|Y_k\} \\ = \ & A_kE\left\{(x_k-E(x_k|Y_k))(x_k-E(x_k|Y_k))^T|Y_k\right\}A_k^T \\ & +\sum_{i=1}^{L}h_i^2(z_k)\Gamma_{A_i}\sigma^2E\{x_kx_k^T|Y_k\}\Gamma_{A_i}^T+R_w \\ = \ & A_k\Sigma_kA_k^T+\Psi_k+R_w \end{aligned}$$

where Ψ_k is defined in (49). Substituting (56) and (57) into (54), we find that

$$\begin{aligned} & \Sigma_{xy|z} \\ = \ & E\{(x_{k+1}-E\{x_{k+1}|Y_k\})[C_k(x_{k+1}-E\{x_{k+1}|Y_k\}) \\ & +\Delta C_kx_{k+1}+v_{k+1}]^T|Y_k\} \\ = \ & E\{(x_{k+1}-E\{x_{k+1}|Y_k\}) \\ & \times(x_{k+1}-E\{x_{k+1}|Y_k\})^TC_k^T|Y_k\} \\ = \ & R_kC_k^T \end{aligned} \tag{58}$$

A similar computation for (55) leads to

$$\begin{aligned}
&\Sigma_{yy|z}\\
=\;& E\{[C_k(x_{k+1}-E\{x_{k+1}|Y_k\})+\Delta C_k x_{k+1}+v_{k+1}]\\
&\times[C_k(x_{k+1}-E\{x_{k+1}|Y_k\})+\Delta C_k x_{k+1}+v_{k+1}]^T|Y_k\}\\
=\;& C_k E\{(x_{k+1}-E\{x_{k+1}|Y_k\})\\
&\times(x_{k+1}-E\{x_{k+1}|Y_k\})^T|Y_k\}C_k^T\\
&+E\{v_{k+1}v_{k+1k}^T|Y_k\}+E\{\Delta C_k x_{k+1}x_{k+1}^T\Delta C_k^T|Y_k\}\\
=\;& C_k R_k C_k^T+R_v\\
&+E\{\sum_{i=1}^{L}\sum_{j=1}^{L}h_i(z_k)h_j(z_k)q_i(k+1)q_j(k+1)\\
&\times\Gamma_{C_i}x_{k+1}x_{k+1}^T\Gamma_{C_j}^T|Y_k\}\\
=\;& C_k R_k C_k^T+R_v\\
&+\sigma^2\sum_{i=1}^{L}h_i^2(z_k)\Gamma_{C_i}E\{x_{k+1}x_{k+1}^T|Y_k\}\Gamma_{C_i}^T\\
=\;& C_k R_k C_k^T+R_v+\sigma^2\sum_{i=1}^{L}h_i^2(z_k)\Gamma_{C_i}\Xi_k\Gamma_{C_i}^T\\
=\;& C_k R_k C_k^T+R_v+\Lambda_k
\end{aligned}\tag{59}$$

where Λ_k is defined in (50). Substituting (58) and (59) into (53), we obtain the recursive equations for $\hat{x}_k$ in (45) and (47). It remains to show (46). By applying (58) and (59) to (6) in Lemma 5 and noting that $\Sigma_{yx|z}=\Sigma_{xy|z}^T$, we have

$$\begin{aligned}
\Sigma_{k+1} &= E\{(x_{k+1}-E\{x_{k+1}|Y_{k+1}\})\\
&\quad\times(x_{k+1}-E\{x_{k+1}|Y_{k+1}\})^T|Y_{k+1}\}\\
&= E\{(x_{k+1}-E(x_{k+1}|Y_k,\ y_{k+1}))\\
&\quad\times(x_{k+1}-E(x_{k+1}|Y_k,\ y_{k+1}))^T|Y_k,\ y_{k+1}\}\\
&= \Sigma_{xx|z}-\Sigma_{xy|z}\Sigma_{yy|z}^{+}\Sigma_{yx|z}\\
&= R_k-R_kC_k^T(C_kR_kC_k^T+R_v+\Lambda_k)^{-1}C_kR_k
\end{aligned}$$

which verifies (46). ■

VII. Simulation Example

In this section, a simulation example is given to confirm the performance of the proposed fuzzy Kalman filter and fuzzy H_∞ filter for the stochastic fuzzy system. Consider the following stochastic T-S fuzzy system:

Rule 1:
IF y_{k-1} is F_{11} and y_{k-2} is F_{21},
THEN

$$\begin{aligned}x_{k+1}&=(A_1+\Delta A_1(k))x_k+B_1u_k+w_k\\ y_k&=(C_1+\Delta C_1(k))x_k+v_k\end{aligned}$$

Rule 2:
IF y_{k-1} is F_{11} and y_{k-2} is F_{22},
THEN

$$\begin{aligned}x_{k+1}&=(A_2+\Delta A_2(k))x_k+B_2u_k+w_k\\ y_k&=(C_2+\Delta C_2(k))x_k+v_k\end{aligned}$$

Rule 3:
IF y_{k-1} is F_{11} and y_{k-2} is F_{23},
THEN

$$\begin{aligned}x_{k+1}&=(A_3+\Delta A_3(k))x_k+B_3u_k+w_k\\ y_k&=(C_3+\Delta C_3(k))x_k+v_k\end{aligned}$$

Rule 4:
IF y_{k-1} is F_{12} and y_{k-2} is F_{21},
THEN

$$\begin{aligned}x_{k+1}&=(A_4+\Delta A_4(k))x_k+B_4u_k+w_k\\ y_k&=(C_4+\Delta C_4(k))x_k+v_k\end{aligned}$$

Rule 5:
IF y_{k-1} is F_{12} and y_{k-2} is F_{22},
THEN

$$\begin{aligned}x_{k+1}&=(A_5+\Delta A_5(k))x_k+B_5u_k+w_k\\ y_k&=(C_5+\Delta C_5(k))x_k+v_k\end{aligned}$$

Rule 6:
IF y_{k-1} is F_{12} and y_{k-2} is F_{23},
THEN

$$\begin{aligned}x_{k+1}&=(A_6+\Delta A_6(k))x_k+B_6u_k+w_k\\ y_k&=(C_6+\Delta C_6(k))x_k+v_k\end{aligned}$$

Rule 7:
IF y_{k-1} is F_{13} and y_{k-2} is F_{21},
THEN

$$\begin{aligned}x_{k+1}&=(A_7+\Delta A_7(k))x_k+B_7u_k+w_k\\ y_k&=(C_7+\Delta C_7(k))x_k+v_k.\end{aligned}$$

Rule 8:
IF y_{k-1} is F_{13} and y_{k-2} is F_{22},
THEN

$$\begin{aligned}x_{k+1}&=(A_8+\Delta A_8(k))x_k+B_8u_k+w_k\\ y_k&=(C_8+\Delta C_8(k))x_k+v_k\end{aligned}$$

Rule 9:
IF y_{k-1} is F_{13} and y_{k-2} is F_{23},
THEN

$$\begin{aligned}x_{k+1}&=(A_9+\Delta A_9(k))x_k+B_9u_k+w_k\\ y_k&=(C_9x+\Delta C_9(k))_k+v_k\end{aligned}$$

The related matrices in the above fuzzy system are defined as follows:

$$\begin{aligned}
&A_1=\begin{bmatrix}0.5&0.3\\0.01&0.6\end{bmatrix},\quad A_2=\begin{bmatrix}0.4&0.7\\0.02&0.5\end{bmatrix},\\
&A_3=\begin{bmatrix}0.3&0.4\\0.03&0.5\end{bmatrix},\quad A_4=\begin{bmatrix}0.2&0.3\\0.04&0.6\end{bmatrix},\\
&A_5=\begin{bmatrix}0.1&0.3\\0.01&0.5\end{bmatrix},\quad A_6=\begin{bmatrix}0.15&0.3\\0.04&0.6\end{bmatrix},\\
&A_7=\begin{bmatrix}0.25&0.3\\0.03&0.5\end{bmatrix},\quad A_8=\begin{bmatrix}0.35&0.3\\0.02&0.5\end{bmatrix},\\
&A_9=\begin{bmatrix}0.45&0.3\\0.01&0.5\end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
&B_1=\begin{bmatrix}1&2\end{bmatrix}^T,\quad B_2=\begin{bmatrix}1&3\end{bmatrix}^T,\quad B_3=\begin{bmatrix}2&1\end{bmatrix}^T,\\
&B_4=\begin{bmatrix}2&3\end{bmatrix}^T,\quad B_5=\begin{bmatrix}1&2\end{bmatrix}^T,\quad B_6=\begin{bmatrix}1&3\end{bmatrix}^T,\\
&B_7=\begin{bmatrix}2&1\end{bmatrix}^T,\quad B_8=\begin{bmatrix}2&3\end{bmatrix}^T,\quad B_9=\begin{bmatrix}1&2\end{bmatrix}^T,
\end{aligned}$$

$$C_1=\begin{bmatrix}1 & 3\end{bmatrix},\quad C_2=\begin{bmatrix}2 & 1\end{bmatrix},\quad C_3=\begin{bmatrix}1 & 2\end{bmatrix},$$
$$C_4=\begin{bmatrix}2 & 3\end{bmatrix},\quad C_5=\begin{bmatrix}2 & 1\end{bmatrix},\quad C_6=\begin{bmatrix}2 & 3\end{bmatrix},$$
$$C_7=\begin{bmatrix}1 & 3\end{bmatrix},\quad C_8=\begin{bmatrix}1 & 2\end{bmatrix},\quad C_9=\begin{bmatrix}2 & 3\end{bmatrix},$$

$$\Gamma_{A_1}=\begin{bmatrix}0.2828 & 0.1414\\ 0.1414 & 0.4243\end{bmatrix},\quad \Gamma_{A_2}=\begin{bmatrix}0.1414 & 0.2828\\ 0.2828 & 0.24243\end{bmatrix},$$
$$\Gamma_{A_3}=\begin{bmatrix}0.2828 & 0.28228\\ 0.1414 & 0.1414\end{bmatrix},\quad \Gamma_{A_4}=\begin{bmatrix}0.1414 & 0.1414\\ 0.4243 & 0.4243\end{bmatrix},$$
$$\Gamma_{A_5}=\begin{bmatrix}0.4243 & 0.2828\\ 0.1414 & 0.4243\end{bmatrix},\quad \Gamma_{A_6}=\begin{bmatrix}0.5657 & 0.1414\\ 0.4243 & 0.2828\end{bmatrix},$$
$$\Gamma_{A_7}=\begin{bmatrix}0.2828 & 0.2828\\ 0.2828 & 0.2828\end{bmatrix},\quad \Gamma_{A_8}=\begin{bmatrix}0.4243 & 0.1414\\ 0.1414 & 0.4243\end{bmatrix},$$
$$\Gamma_{A_9}=\begin{bmatrix}0.2828 & 0.4243\\ 0.5657 & 0.5657\end{bmatrix},$$

$$\Gamma_{C_1}=\begin{bmatrix}0.0424 & 0.0283\end{bmatrix},\quad \Gamma_{C_2}=\begin{bmatrix}0.0141 & 0.0283\end{bmatrix},$$
$$\Gamma_{C_3}=\begin{bmatrix}0.0566 & 0.0283\end{bmatrix},\quad \Gamma_{C_4}=\begin{bmatrix}0.0141 & 0.0354\end{bmatrix},$$
$$\Gamma_{C_5}=\begin{bmatrix}0.0212 & 0.0283\end{bmatrix},\quad \Gamma_{C_6}=\begin{bmatrix}0.0495 & 0.0169\end{bmatrix},$$
$$\Gamma_{C_7}=\begin{bmatrix}0.0141 & 0.0707\end{bmatrix},\quad \Gamma_{C_8}=\begin{bmatrix}0.0198 & 0.0283\end{bmatrix},$$
$$\Gamma_{C_9}=\begin{bmatrix}0.0849 & 0.0283\end{bmatrix},$$

The premise variables are chosen as y_{k-1} as well as y_{k-2} and the membership functions for these premise variables are given in Fig. 1.

The input signal is chosen as $u(t) = 5\sin(t)$, while noises w_k and v_k are zero-mean Gaussian white noise with $R_w = \begin{bmatrix}0.25 & 0\\ 0 & 0.25\end{bmatrix}$ and $R_v = 0.25$, respectively. The white process $q_i(k)$ is zero-mean with variance $\sigma^2 = 0.02$. The initial condition of the sate $x(k)$ is given by

$$\begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix}=\begin{bmatrix}10\\ 10\end{bmatrix}$$

The initial conditions of all the estimators in the simulation study are all set as

$$\begin{bmatrix}\hat{x}_1(0)\\ \hat{x}_2(0)\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

1) Conventional State Observer Design: For the standard state observer given in (14), the main work is to solve the LMI (18) to find the local observer gain matrix L_i for the i-th rule of the fuzzy observer. We set $\lambda_1 = 0.8$, $\lambda_2 = 0.8$, and Q_1 as well as Q_2 are both identity matrices. We solve the linear matrix inequality (18) by using the Matlab LMI Toolbox to obtain the observer gain matrices as

$$L_1=\begin{bmatrix}0.1677\\ 0.1610\end{bmatrix}\quad L_2=\begin{bmatrix}0.2594\\ 0.1378\end{bmatrix}\quad L_3=\begin{bmatrix}0.1601\\ 0.1386\end{bmatrix}$$
$$L_4=\begin{bmatrix}0.1152\\ 0.1660\end{bmatrix}\quad L_5=\begin{bmatrix}0.0937\\ 0.1341\end{bmatrix}\quad L_6=\begin{bmatrix}0.1062\\ 0.1658\end{bmatrix}$$
$$L_7=\begin{bmatrix}0.1244\\ 0.1383\end{bmatrix}\quad L_8=\begin{bmatrix}0.1420\\ 0.1367\end{bmatrix}\quad L_9=\begin{bmatrix}0.1594\\ 0.1350\end{bmatrix}$$

Note that the standard state observer given in (14) is a prediction-type estimator given by

$$\hat{x}_{k+1} = A_k\hat{x}_k + B_k u_k + \sum_{i=1}^{9} h_i(z_k) L_i (y_k - C_{k-1}\hat{x}_k) \quad (60)$$

With the same observer gain matrices L_i, we can also construct a filtering-type estimator as follows

$$\begin{aligned}\hat{x}_{k+1} &= A_k\hat{x}_k + B_k u_k\\ &\quad + \sum_{i=1}^{9} h_i(z_k) L_i (y_{k+1} - C_k A_k \hat{x}_k - C_k B_k u_k)\end{aligned} \quad (61)$$

which is expected to have better estimation performance than the prediction-type estimator as the additional information y_{k+1} is used.

2) Optimal H_∞ Filter Design: For the optimal H_∞ filter design, we solve the LMI's in (32) and (33). The minimal value of ρ is $\rho = 5.9648$,

$$P_1=\begin{bmatrix}2.6506 & 2.0855\\ 2.0855 & 15.8677\end{bmatrix}\quad P_2=\begin{bmatrix}1.6453 & -0.4782\\ -0.4782 & 1.7285\end{bmatrix},$$

and the observer gain matrices are

$$L_1=\begin{bmatrix}0.1947\\ 0.1376\end{bmatrix}\quad L_2=\begin{bmatrix}0.2861\\ 0.1479\end{bmatrix}\quad L_3=\begin{bmatrix}0.1747\\ 0.1213\end{bmatrix}$$
$$L_4=\begin{bmatrix}0.1649\\ 0.1356\end{bmatrix}\quad L_5=\begin{bmatrix}0.1008\\ 0.1113\end{bmatrix}\quad L_6=\begin{bmatrix}0.1627\\ 0.1355\end{bmatrix}$$
$$L_7=\begin{bmatrix}0.1360\\ 0.1170\end{bmatrix}\quad L_8=\begin{bmatrix}0.1589\\ 0.1179\end{bmatrix}\quad L_9=\begin{bmatrix}0.1388\\ 0.0974\end{bmatrix}$$

Note that the considered optimal H_∞ filter in Section V is of prediction type. As discussed in the previous section, we can also construct a filtering-type estimator related to the optimal H_∞ filter.

3) The Optimal Fuzzy Kalman Filter: We shall simulate both the filtering-type optimal fuzzy Kalman filter in (45)-(50) and the prediction-type one in (52). The initial condition of the conditional covariance matrix Σ_k is given by

$$\Sigma_0 = 2\times 10^3 \times \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

4) Comparison of Estimation Performance: The standard fuzzy estimator, the optimal H_∞ fuzzy filter, and the optimal Kalman fuzzy filter of both prediction type and filtering type will be compared by verifying the standard deviation σ_x of the state estimation error by counting 10000 sample points. For the three estimators of filtering type, estimations of $x_1(t)$ and $x_2(t)$ of the stochastic T-S fuzzy system are shown in Fig. 2 (a) and Fig.3 (a), respectively. The related estimation errors are shown in Fig. 2(b) and Fig. 3(b). While for the three estimators of prediction type, estimations of $x_1(t)$ and $x_2(t)$ are shown in Fig. 4 and Fig. 5, respectively. The standard deviations of estimation errors of these estimators are compared in Table I. In this table, it is shown that all the filtering-type estimators outperform the prediction-type ones. In the class of filtering-type estimators, the standard deviation of the estimation error for the optimal Kalman fuzzy filter is much less than those of the other two filters. However, the performances of the filtering-type estimators are very close.

Now we turn to evaluate the robustness of the three estimators. Note that the designs of the standard fuzzy estimator and the H_∞ fuzzy optimal estimator are irrelevant to R_v and R_w. Here, the optimal Kalman fuzzy filter will be computed with the covariance matrices R_v and R_w given above. However, in the simulation of the system responses, we use different settings of these two covariance matrices, including $(R_{v_1} = 4R_v, R_{w_1} = 4R_w)$, $(R_{v_2} = 8R_v, R_{w_2} = 8R_w)$, and $(R_{v_3} = 16R_v, R_{w_3} = 16R_w)$. The standard deviations of the state estimation errors for various state estimators under different settings are compared in Table II. It is surprising to find that the optimal Kalman fuzzy filter derived in (45)-(50) has the most robust performance with the smallest standard deviation of the state estimation error, although there are large uncertainties of the noise covariance matrices in implementing the filter.

VIII. Conclusion and Discussion

In this study, the state estimation problem for the stochastic T-S fuzzy model with state-dependent noise on the system matrix and the output matrix has been attacked. First, we have derived sufficient conditions for a class of standard fuzzy state observer to ensure that the state estimation error is mean square bounded. The observer gain matrices in the fuzzy observer can be obtained by solving a linear matrix inequality. Then, the optimal H_∞ fuzzy filtering problem is considered to minimize the worst-case ratio of the power of the state estimation error to that of the external noises. The optimal H_∞ observer gain matrices can be obtained by solving two linear matrix inequalities. To further improve estimation performance, we have studied the optimal Kalman fuzzy filtering problem with the known statistical information of the process noise and the measurement noise of the uncertain stochastic T-S fuzzy model. It is shown that the minimum-variance estimation for the uncertain stochastic T-S fuzzy model is actually a linear estimation problem from the viewpoint of conditional expectation. Actually, The structure of the developed optimal Kalman fuzzy filter also very resembles that of the conventional Kalman filter. Comparison of estimation performances of the developed three estimators is made via simulation study which verifies the optimal and robust performance of the optimal Kalman fuzzy filter.

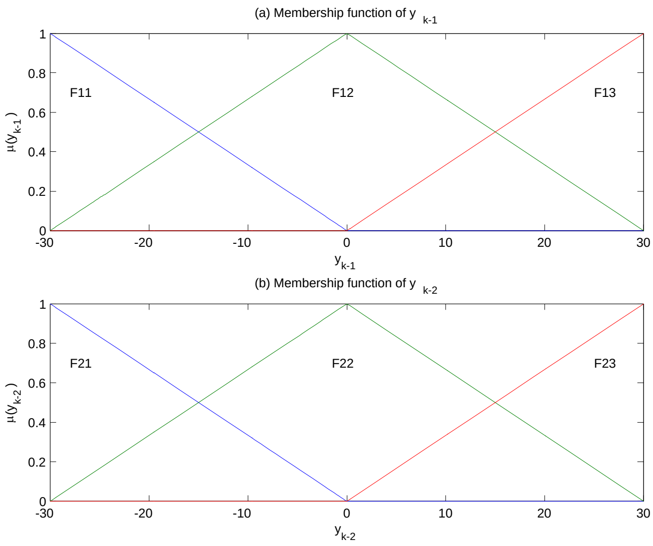

Fig. 1. Membership functions of the premise variables.

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	Kalman Filtering	Standard Filtering	H_∞ Filtering
σ_x	2.9796	9.8979	13.3157
	Kalman Pred.	Standard Pred.	H_∞ Pred.
σ_x	26.4816	26.6690	26.5136

TABLE I

COMPARISON OF STANDARD DEVIATIONS OF ESTIMATION ERRORS OF THE THREE ESTIMATOR OF BOTH THE FILTERING TYPE AND THE PREDICTION TYPE.

R_v/R_w	Kalman Filtering	Standard Filtering	H_∞ Filtering
R_{v1}/R_{w1}	3.9955	10.6300	13.7755
R_{v2}/R_{w2}	4.5206	11.7857	15.6592
R_{v3}/R_{w3}	4.9618	13.2356	17.4602
R_v/R_w	Kalman Pred.	Standard Pred.	H_∞ Pred.
R_{v1}/R_{w1}	28.4393	28.8041	26.5136
R_{v2}/R_{w2}	31.0104	31.1080	31.1867
R_{v3}/R_{w3}	35.3288	35.7794	35.2731

TABLE II

STANDARD DEVIATIONS OF ESTIMATION ERRORS UNDER DIFFERENT SETTINGS BY VARYING THE COVARIANCE MATRICES R_v AND R_w.

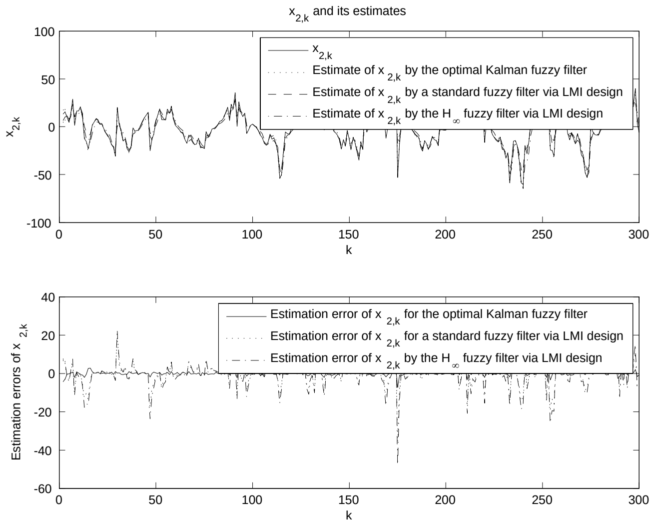

Fig. 3. (a) $x_2(t)$ and its estimation (b) Estimation error of $x_2(t)$. (Optimal Kalman fuzzy filter,standard fuzzy estimator and the optimal H_∞ fuzzy filter (61))

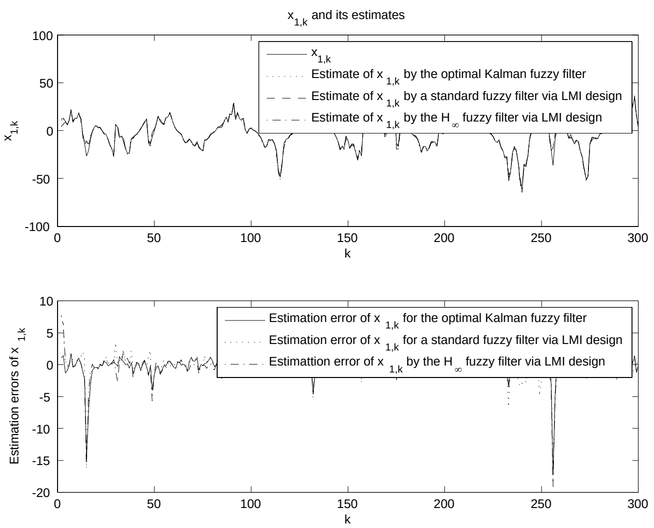

Fig. 2. (a) $x_1(t)$ and its estimation (b) Estimation error of $x_1(t)$. (Optimal Kalman fuzzy filter,standard fuzzy estimator and the optimal H_∞ fuzzy filter (61))

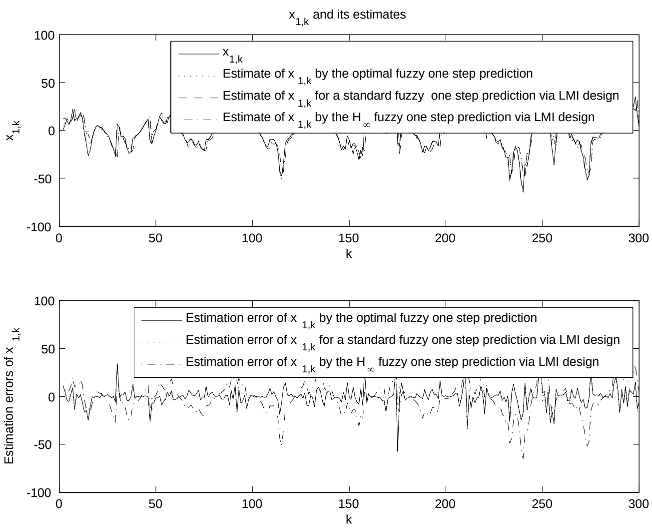

Fig. 4. (a) $x_1(t)$ and its estimation (b) Estimation error of $x_1(t)$. (Optimal fuzzy one-step ahead predictor,standard fuzzy one-step ahead predictor and the optimal H_∞ fuzzy one-step ahead predictor (60)via)

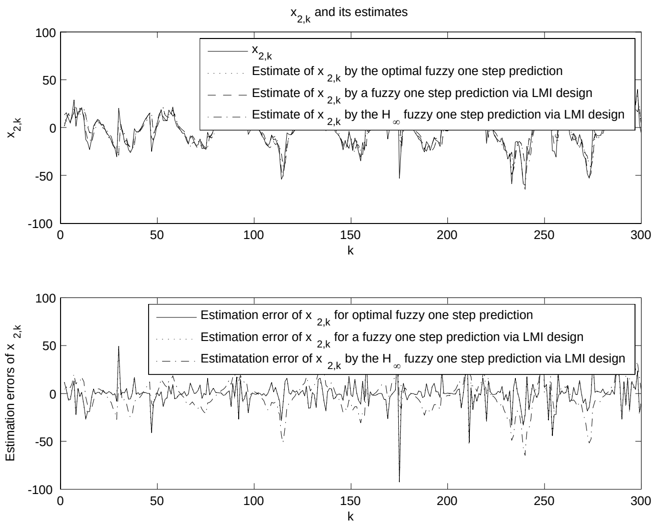

Fig. 5. (a) $x_2(t)$ and its estimation (b) Estimation error of $x_2(t)$. (Optimal fuzzy one-step ahead predictor,standard fuzzy one-step ahead predictor and the optimal H_∞ fuzzy one-step ahead predictor (60)via)

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附件三

報告人姓名	李柏坤	服務機構 及職稱	中華大學電機系 教授
時間 會議 地點	96 年 3 月 21 日至 96 年 3 月 23 日 香港	本會核定 補助文號	國科會計畫 95-2221-E-216-034 中核定出席國際會議經費六萬元
會議 名稱	（中文）2007 年工程與計算機科學聯合國際會議 (IMECS 2007) （英文）The International MultiConference of Engineers and Computer Scientists 2007 (IMECS 2007)		
發表 論文 題目	1.（中文）使用 PID 類型的學習法則的適應模糊控制器 （英文）Adaptive Fuzzy Control using PID-Type Learning Algorithm 2.（中文）適應性增長與刪除類神經控制器運用於線型陶瓷伺服馬達 （英文）Design of Adaptive Growing-And-Pruning Neural Control for LPCM Drive System		

報告內容應包括下列各項：

一、參加會議經過

此次 International Association of Engineers 舉辦的 The International MultiConference of Engineers and Computer Scientists 2007 (IMECS 2007)，整個聯合研討會包含

[1] International Conference on Artificial Intelligence and Applications (ICAIA)、
[2] International Conference on Bioinformatics (ICB)、
[3] International Conference on Control and Automation (ICCA)、
[4] International Conference on Computer Science (ICCS)、
[5] International Conference on Communication Systems and Applications (ICCSA)、
[6] International Conference on Data Mining and Applications (ICDMA)、
[7] International Conference on Electrical Engineering (ICEE)、
[8] International Conference on Imaging Engineering (ICIE)、
[9] International Conference on Industrial Engineering (ICINDE)、
[10] International Conference on Internet Computing and Web Services (ICICWS)、
[11] International Conference on Operations Research (ICOR)、
[12] International Conference on Scientific Computing (ICSC) 、
[13] International Conference on Software Engineering (ICSE)。

此次 IMECS 2007 聯合研討會，於 96 年 3 月 21 日到 96 年 3 月 23 日，在香港 Regal Kowloon Hotel 舉行。本人在 3 月 20 上午 10:20 自中正機場乘坐班機前往香港。

本人於 ICCA 研討會發表發表兩篇論文，分別是

(1) Adaptive Fuzzy Control using PID-Type Learning Algorithm，於 3 月 21 日 10:45 的 ICCA II Session 發表。

(2) Design of Adaptive Growing-And-Pruning Neural Control for LPCM Drive System，於 3 月 21 日 16:00 的 ICCA IV Session 發表。

上面兩篇文章也被推薦競選 Best Paper Award。

同時我也積極參與 International Conference on Artificial Intelligence and Applications (ICAIA)、International Conference on Bioinformatics (ICB)以及 International Conference on Electrical Engineering (ICEE)等研討會。其中並參加 3 月 22 日，林志民教授(Prof. Chih-Min Lin)於 ICCA 研討會之專題演講(ICCA 2007 Invited Talk)，演講題目為 Design and Application of Adaptive Cerebellar Model Articulator Controller。

希望藉由此 IMECS 2007 聯合研討會與讓世界各國之研究團隊切磋，提升國際間學術交流，互相激勵研究靈感，以求在研究上更進一步之突破。

3 月 23 日，於結束我參加在 ICCA V Session 之研討會報告，即準備 Check out，前往機場搭乘 17:05 由香港起飛的班機返回台灣，結束這次的 IMECS 2007 國際會議之行。

二、與會心得

(1) 從此次研討會中認識了很多來自全世界各地的菁英學者，是此次最大的收穫，對於將來推動國際學術交流，有相當大的幫助。

(2) 從此次研討會所發表的論文來看，各國有關智慧型控制系統發展都有顯著的研究成果，國內於學理分析方面算相當不錯，但於系統整合與應用則有待改善。

(3) 智慧型控制理論應用於生物資訊，也受到全世界的重視，國內應該即起直追。

三、考察參觀活動(無是項活動者省略)

主辦單位無舉辦任何考察參觀活動。

四、建議

(1) 於最近才成立的 International Association of Engineers，非常積極的舉辦國際聯合研討會，使得全世界各地的菁英學者，能夠共聚一堂。我覺得國內也可以由幾個學會或中國工程師協會，舉辦國際聯合研討會。

(2) 香港的交通建設與觀光產業有高度的結合，使得舉辦國際研討會有相當好的基礎，這一點是我們還比不上的，要提升台灣的整體學術地位，舉辦國際聯合研討會，促進學術交流，是必要的一步，我們在交通建設與觀光產業的結合，還需要加油。

(3) 香港的國際化，已經非常根深蒂固，同時以此為基礎邁向國際之競爭，與香港相比，台灣的自由化與國際化還有待大家的努力。

五、攜回資料名稱及內容

(1) 完整論文光碟片。

(2) 論文摘要紙本。

六、其他

無。