行政院國家科學委員會專題研究計畫 成果報告

非線性隨機控制理論研究及其在非傳統控制領域的應用--子計畫二：非線性隨機模糊適應控制與其在無線網路之應用(II)

研究成果報告(精簡版)

計 畫 類 別 ：整合型
計 畫 編 號 ：NSC 96-2221-E-216-036-
執 行 期 間 ：96年08月01日至97年07月31日
執 行 單 位 ：中華大學電機工程研究所

計畫主持人：李柏坤

計畫參與人員：碩士班研究生-兼任助理人員：林炫亨

報 告 附 件 ：出席國際會議研究心得報告及發表論文

處 理 方 式 ：本計畫可公開查詢

中 華 民 國 97年10月03日

『非線性隨機控制理論研究及其在非傳統控制領域的應用』子計畫二:
非線性隨機模糊適應控制與其在無線網路之應用(II)
Fuzzy Adaptive Control of Nonlinear Stochastic System and Its Applications to Wireless Network (II)

計畫編號:NSC-96-2221-E-216-036
執行期限:96年8月1日至97年7月31日
主持人: 李柏坤 教授 中華大學電機系

一、中文摘要

在此研究中, 我們已完成隨機 T-S 模糊 ARMAX 模式之適應最小變異量控制。對於一個隨機 T-S 模糊 ARMAX 模式, 首先我們推導其最佳向前一步的預估模式; 基於此預估模式, 我們使用隨機梯度方法來估測其中之參數。接著於採用之直接適應控制結構下, 我們推導其適應控制律, 使得針對一個參考模式的輸出追蹤誤差的變異量能夠最小化。我們推導此適應控制系統的穩定度及其性能分析, 同時藉由模擬研究已驗證所推導之理論。

關鍵詞: 模糊適應控制、隨機T-S 模糊 ARMAX 模式

Abstract

Adaptive minimum variance control for stochastic T-S fuzzy ARMAX model is addressed in this study. From the fuzzy ARMAX model, a fuzzy one-step ahead prediction model is first introduced. A stochastic gradient algorithm is then proposed to identify the parameters of the related one-step-ahead predictor. Under the direct adaptive control scheme, minimum variance control is applied to find the control law to make the output track a desired reference signal. Stability and performance of the adaptive stochastic fuzzy control system are rigorously derived. Simulation study is also made to verify the developed results.

Keywords: Fuzzy adaptive control, Stochastic T-S fuzzy ARMAX model

二、緣由與目的

Recently, based on the Takagi-Sugeno model, fuzzy modeling for nonlinear dynamic systems and identification problem are discussed in [1]-[3]. Meanwhile, fuzzy control scheme has been employed for tracking control of nonlinear systems based on the adaptive feedback linearization techniques [4]-[8]. In the previously mentioned literature, the external disturbances or noises are considered to be deterministic for the convenience of control design. However, in many practical applications [9][10], external noises are inevitable and are more adequately described by random processes. In this situation, the systems to be controlled are always modeled by stochastic systems. A nonlinear stochastic system can be approximated by a fuzzy stochastic system [11]-[16]. However, it is more difficult to design a control law to achieve the optimal tracking of fuzzy stochastic systems because the membership functions of the fuzzy stochastic system are also functions of the random premise variables. This will make the identification problem and the control design of the stochastic fuzzy systems more difficult and complicated.

Up to date, the stochastic fuzzy modeling and adaptive control issues are seldom addressed in the literature. A stochastic adaptive control scheme for the state-space T-S fuzzy model based on the LQG control theory is proposed in [17]. Non-adaptive LQG fuzzy controllers are also considered in [11] and [12]. On the other hand, the NARMAX (nonlinear ARMAX) model has been presented for modeling nonlinear processes. The NARMAX model can be reduced to a quasi-ARMAX system by linearization or approximation. Fuzzy system identification and nonlinear model predictive control based on the quasi-ARMAX model are discussed in [13][14][15]. Besides the quasi-ARMAX model, the fuzzy ARMAX model has been used to forecast the short-term load of a power system in [16]. However, these algorithms proposed by the above mentioned literature are given without vigorous proofs.

Adaptive minimum variance control for stochastic T-S fuzzy ARMAX model will be addressed in this study. From the fuzzy ARMAX model, a fuzzy one-step ahead prediction model will be first introduced. A stochastic gradient algorithm will then be proposed to identify the parameters of the related one-step-ahead predictor. Under the direct adaptive control scheme, minimum variance control will be applied to find the control law to make the output track a desired reference signal. Stability and performance of the adaptive stochastic fuzzy control system will be rigorously derived.

The remainder of this study is organized as follows. System description and problem formulation for the identification of the fuzzy ARMAX model are described in Section 3.1. General stochastic stability results of the T-S fuzzy model are attacked in Section 3.2. Then the one-step ahead predictor for the fuzzy ARMAX model is introduced in Section 3.3. Based on the developed predictor, a stochastic gradient algorithm, together with the parameter convergence properties, for identifying the parameters of the optimal one-step ahead predictor are given in Section 3.4. Adaptive minimum variance control design is discussed in Section 3.5. Stability and tracking performance of the adaptive minimum variance fuzzy control system are proved in Section 3.6. Simulation study is discussed in Section 3.7. Conclusions and discussions are given in Section **??**.

Notations and Definitions

Let $\|x\|$ be the Euclidean norm of a vector x. Let $A(q^{-1})$ be a polynomial with $A(q^{-1}) = \sum_{i=0}^{n} a_i q^{-i}$. The companion matrix Ξ_A associated with the polynomial $A(q^{-1})$ is defined as

$$\Xi_A = \begin{bmatrix} 0_{(n-1)\times 1} & I_{n-1} & & \\ -a_n & -a_{n-1} & \cdots & -a_1 \end{bmatrix}$$

三、研究方法及成果

A. System modeling and problem formulation

The i-th rule of the considered stochastic fuzzy T-S ARMAX model is given by:

Plant Rule i:

$$\begin{array}{ll} \text{If} & z_1(k) \text{ is } F_{i1} \text{ and } z_2(k) \text{ is } F_{i2} \\ & \text{and } \cdots\cdots \text{and } z_{g_0}(k) \text{ is } F_{ig_0} \\ \text{Then} & A_i(q^{-1})y(k+1) \\ & = B_i(q^{-1})u(k) + C_i(q^{-1})w(k+1) \end{array} \quad (1)$$

for i = 1, 2,..., L, where F_{ij} is the fuzzy set, $z_1(k)$, $z_2(k)$, $\cdots$, $z_{g_0}(k)$ are the premise variables, and L is the number of if-then rules. Polynomials $A_i(q^{-1}), B_i(q^{-1})$, and $C_i(q^{-1})$ are defined, respectively, as follows

$$\begin{array}{ll} A_i(q^{-1}) = a_{i0} + a_{i1}q^{-1} + ... + a_{in}q^{-n}, & a_{i0} = 1 \\ B_i(q^{-1}) = b_{i0} + b_{i1}q^{-1} + ... + b_{im}q^{-m}, & \\ C_i(q^{-1}) = c_{i0} + c_{i1}q^{-1} + c_{i2}q^{-2} + ... + c_{il}q^{-l}, & c_{i0} = 1 \end{array} \quad (2)$$

for i = 1, 2,..., L, where q^{-1} denotes the delay operator, i.e., $q^{-1}y(k) = y(k-1)$. Without loss of generality, $C_i(q^{-1})$ can be taken to have roots inside the unit circle [9][10]. $y(k)$ is the output measurement, $u(k)$ the control input, and the noise process $w(k)$ will be taken to satisfy the following assumptions [9][10]:

$$E[w(k+1)|\ \digamma_k] = 0,\ a.\ s. \quad (3)$$

$$E[w^2(k+1)|\ \digamma_k] = \sigma_w^2,\ a.\ s. \quad (4)$$

$$\limsup_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} w^2(k) \le K_w < \infty,\ a.\ s. \quad (5)$$

where E denotes the expectation, $\digamma_k$ denotes the sub-σ algebra generated from the data set $\{y(s)\}_{s\le k}$. Note that $\digamma_k$ is increasing, i.e., $\digamma_k \subset \digamma_{k+1}$. We shall demand that $u(k)$ is $\digamma_k-$measurable. For the premise variables $z_i(k), 1 \le i \le g_0$, we assume that they are $\digamma_k-$measurable, i.e., $z_i(k)$ depends on the data set $\{y(s), u(s)\}_{s\le k}$. Using the smoothing property of the conditional mean [18], conditions (3) and (4) imply that $w(k)$ is also a white process with zero mean and variance σ_w^2. Note that condition (5) implies

$$\frac{1}{N}\sum_{k=1}^{N} w^2(k) \le K_w,\ a.s., \text{ for } N \ge N_w \quad (6)$$

where N_w is a sufficiently large integer.

Given the input/output sequences $\{u(k)\}$ and $\{y(k)\}$, the stochastic fuzzy system (1) is equivalent to

$$\begin{aligned} y(k+1) = \sum_{i=1}^{L} h_i(z(k))\{(1 - A_i(q^{-1}))y(k+1) \\ + B_i(q^{-1})u(k) + C_i(q^{-1})w(k+1)\} \end{aligned} \quad (7)$$

where $z(k) = [z_1(k)\ z_2(k)\ ...\ z_{g_0}(k)]$ and, for $1 \le i \le L$,

$$\mu_i(z(k)) = \prod_{j=1}^{g_0} F_{ij}(z_j(k)) \quad (8)$$

$$h_i(z(k)) = \frac{\mu_i(z(k))}{\sum_{i=1}^{L}\mu_i(z(k))} \quad (9)$$

where the function $F_{ij}(z_j(k))$ is the grade of membership of $z_j(k)$ in F_{ij}. For (8) and (9), we assume that

$$h_i(z(k)) \ge 0\ ,\ \sum_{i=1}^{L} h_i(z(k)) = 1 \quad (10)$$

The physical meaning of (7) is that the L local linear stochastic subsystems are interpolated by the fuzzy basis functions $h_i(z(k))$, for $i = 1, 2, \ldots, L$.

In the sequel, we shall first attack the identification problem for estimating the parameters of the optimal predictor related to the fuzzy ARMAX model (1). After obtaining the estimates of the parameters, the design objective is to determine the adaptive control input $u(k)$, as a function of $\{y(s), u(s-1)\}_{s\le k}$, to minimize the mean square error [10]

$$J_1(k+1) = E\{[y(k+1) - y^*(k+1)]^2|\digamma_k\} \quad (11)$$

between the the output $y(k+1)$ and the bounded reference signal $y^*(k+1)$.

B. Stability of Stochastic T-S Fuzzy Systems

In order to deal with the adaptive control problem of the stochastic T-S fuzzy ARMAX model, the stability issue of the stochastic fuzzy system must be addressed first. Since the fuzzy ARMAX model, such as in (7), can be transformed into a state-space stochastic fuzzy model and stability is easier to discuss from the state-space perspective, we consider a forced T-S fuzzy system in the state-space form as follows

$$\begin{array}{rcl} x(k+1) & = & [A(k)x(k) + B(k)u_s(k)] \\ y_s(k) & = & [C(k)x(k) + D(k)u_s(k)] \end{array} \quad (12)$$

where the sequences $\left\{\|A(k)\|^2\right\}$, $\left\{\|B(k)\|^2\right\}$, $\left\{\|C(k)\|^2\right\}$, and $\left\{\|D(k)\|^2\right\}$ are uniformly bounded. It is also assumed that $A(k)$, $B(k)$, $C(k)$, and $D(k)$ are all $\digamma_k-$measurable.

Theorem 1: If there exists a sequence of symmetric positive definite matrices $\{P(k)\}$ with $0 < \lambda_P^{\min} I \le P(k) \le \lambda_P^{\max} I <$

∞ and $P(k)$ being $\digamma_k-$measurable such that the matrix inequality

$$\lambda P(k) - A^T(k) E\left\{P(k+1)|\digamma_k\right\} A(k) > 0, \ \forall k \tag{13}$$

holds for some λ with $0 < \lambda < 1$, then the stochastic fuzzy system

$$x(k+1) = A(k)x(k) \tag{14}$$

is exponentially stable in the sense that

$$\|x(k)\| \leq c_1(\sqrt{\lambda})^{k-k_0} \|x(k_0)\|, \quad k \geq K_1, \ a.s. \tag{15}$$

for some positive almost surely bounded random variable $c_1 > 0$ and a sufficiently large integer K_1.

Proof: For the convenience of review process, the proof is given in Appendix A. ■

Corollary 1: With the same condition given in Theorem 1, the transition matrix $\Phi(k+1, k_0)$, defined as

$$\Phi(k+1, k_0) \triangleq A(k)A(k-1)\cdots A(k_0) \tag{16}$$

with $\Phi(k,k) \triangleq I$, has an upper bound of the induced norm of $\Phi(k, k_0)$ in the almost sure sense as

$$\|\Phi(k, k_0)\| \leq c_2(\sqrt{\lambda})^{k-k_0}, \quad k \geq K_1, \ a.s. \tag{17}$$

for some positive almost surly bounded random variable c_2 and a sufficiently large integer K_1.

With the help of the above theorem, we can obtain the main stability result for further analysis of global stability and tracking performance of the proposed adaptive fuzzy minimum variance control system.

Theorem 2: For the stochastic system in (12), there exists a sequence of symmetric positive definite matrices $\{P(k)\}$ with $0 < \lambda_P^{\min} I \leq P(k) \leq \lambda_P^{\max} I < \infty$ and $P(k)$ being $\digamma_k-$measurable such that the matrix inequality (13) hold for some λ with $0 < \lambda < 1$, then we have, for $N \geq K_1$,

$$\frac{1}{N}\sum_{k=1}^{N} \|y_s(k)\|^2 \leq \frac{K_2}{N}\sum_{k=1}^{N} \|u_s(k)\|^2 + \frac{K_3}{N}, a.\ s. \tag{18}$$

where K_1 is a sufficiently large number, $0 < K_2 < \infty$, and $0 \leq K_3 < \infty$.

Proof: The proof is given in Appendix B. ■

C. Optimal predictor of stochastic fuzzy systems

In this section, the prediction problem of the fuzzy ARMAX model in (7) will be discussed. This will result in a fuzzy predictor model which will be suitable for parameter estimation and direct adaptive tracking control design for the fuzzy ARMAX model. The optimal fuzzy predictor for the fuzzy ARMAX model has been studied in our previous study [19]. Some related results in that reference are briefly summarized in the following.

Assumption 1: Let $\Xi_{C,i}$ be the companion matrix associated with the polynomial $C_i(q^{-1})$. Assume that there exist symmetric positive matrices $P_{C,i}, \ 1 \leq i \leq L$, such that the set of matrix inequalities

$$\begin{bmatrix} \lambda_C P_{C,i} & \Xi_{C,i}^T P_{C,j} \\ P_{C,j}\Xi_{C,i} & P_{C,j} \end{bmatrix} > 0, \ 1 \leq i, \ j \leq L \tag{19}$$

is solvable for some λ_C with $0 < \lambda_C < 1$.

Let $y^0(k+1|k)$ denote the conditional mean of $y(k+1)$ given the data set $\{u(s), \ y(s)\}_{s\leq k}$, i.e., $y^0(k+1| \ k) \triangleq E\left\{y(k+1)| \ \digamma_k\right\}$. Define the polynomial $\alpha_i(q^{-1}), \ 1 \leq i \leq L$, as

$$C_i(q^{-1}) - A_i(q^{-1}) = q^{-1}\alpha_i(q^{-1}) \tag{20}$$

where

$$\alpha_i(q^{-1}) = \alpha_{i0} + \alpha_{i1}q^{-1} + \cdots + q^{-(\overline{n}-1)}, \ \overline{n} = \max(n, \ l)$$

Under **Assumption 1** on the fuzzy ARMAX model (7), the optimal one-step ahead predictor of $y(k+1)$ given the data set $\{u(s), \ y(s)\}_{s\leq k}$ is $y^0(k+1| \ k)$ which satisfies the following equation

$$\begin{aligned} y^0(k+1|k) = \sum_{i=1}^{L} h_i(z(k))\{\left[1 - C_i(q^{-1})\right] y^0(k+1|k) \\ + \alpha_i(q^{-1})y(k) + B_i(q^{-1})u(k)\} \end{aligned} \tag{21}$$

with the prediction error

$$y(k+1) - y^0(k+1|k) = w(k+1) \tag{22}$$

Equation (21) defines a unique *fuzzy prediction model* corresponding to the fuzzy ARMAX model (7).

D. Stochastic Gradient Algorithm

Following from the fuzzy prediction model represented by (21), the stochastic gradient algorithm in [10] will be used to identify the parameters. First, rearrange the prediction model (21) as follows

$$y^0(k+1|k) = \sum_{i=1}^{L} h_i(z(k))\chi_0^T(k)\theta_{i0} = \phi_0^T(k)\theta_0 \tag{23}$$

where

$$\begin{aligned} \chi_0(k) &= \left[-y^0(k|k-1)\cdots - y^0(k-l+1|k-l)\right. \\ &\quad \left. y(k)\cdots y(k-\overline{n}+1) \ u(k)\cdots u(k-m)\right]^T \\ \theta_{i0} &= \left[c_{i1}\cdots c_{il} \ \alpha_{i0}\cdots\alpha_{i(\overline{n}-1)} \ b_{i0}\cdots b_{im}\right]^T, \ 1 \leq i \leq L \\ \phi_0(k) &= \left[h_1(z(k))\chi_0^T(k) \quad h_2(z(k))\chi_0^T(k)\right. \\ &\quad \left.\cdots\cdots \quad h_L(z(k))\chi_0^T(k)\right]^T \\ \theta_0 &= \left[\theta_{10}^T \quad \theta_{20}^T \quad \cdots\cdots \theta_{L0}^T\right]^T \end{aligned} \tag{24}$$

Note that (23) represents a pseudo linear regression form for the fuzzy ARMAX prediction model (21) because the component $y^0(k-i+1|k-i)$ in $\chi_0(k)$ depends on the true parameter vector θ_0. According to the pseudo linear regression form (23), the proposed stochastic gradient algorithm to identify the true parameter vector θ_0 is given by, for $k \geq 1$,

$$\begin{aligned} \widehat{\theta}(k) = \widehat{\theta}(k-1) + \frac{\phi(k-1)}{r(k-2) + \phi^T(k-1)\phi(k-1)} \\ \times \left[y(k) - \phi^T(k-1)\widehat{\theta}(k-1)\right] \end{aligned} \tag{25}$$

where the regression vector $\phi(k)$ and the function $r(k)$ are defined as

$$\phi(k) = \left[h_1(z(k))\chi^T(k) \quad h_2(z(k))\chi^T(k) \right. \\ \left. \cdots\cdots \quad h_L(z(k))\chi^T(k)\right]^T \tag{26}$$

$$\chi(k) = [-\overline{y}(k) \cdots - \overline{y}(k-l+1)\ y(k) \cdots y(k-\overline{n}+1) \\ u(k) \cdots u(k-m)]^T \tag{27}$$

$$\overline{y}(k) = \phi^T(k-1)\widehat{\theta}(k) \tag{28}$$

$$r(k-1) = r(k-2) + \phi^T(k-1)\phi(k-1) \tag{29}$$

For the initial conditions, $\widehat{\theta}(0)$ can be arbitrarily chosen and $r(-1)$ must be a positive scalar. By its definition, the variable $\overline{y}(k)$ can be regarded as a posterior estimate of $y(k)$.

Before proceeding to analyze the stochastic gradient algorithm, some useful definitions are made as follows

$$\widehat{y}(k) = \phi^T(k-1)\widehat{\theta}(k-1) \tag{30}$$

$$e(k) = y(k) - \widehat{y}(k) \tag{31}$$

$$\eta(k) = y(k) - \overline{y}(k) \tag{32}$$

$$\varsigma(k) = \eta(k) - w(k) \tag{33}$$

$$\widetilde{\theta}(k) = \widehat{\theta}(k) - \theta_0 \tag{34}$$

$$\beta(k) = -\phi^T(k-1)\widetilde{\theta}(k) \tag{35}$$

Some general properties of the stochastic gradient algorithm can be extracted from Lemma 8.5.1 in [10] as follows.

$$\lim_{N\to\infty}\sum_{k=1}^{N}\frac{\phi^T(k-1)\phi(k-1)}{r(k-1)r(k-2)} < \infty \tag{36}$$

$$\eta(k) = \frac{r(k-2)}{r(k-1)}e(k) \tag{37}$$

$$E\left\{\beta(k)w(k)|\ F_{k-1}\right\} = \\ -\frac{\phi^T(k-1)\phi(k-1)}{r(k-1)}\sigma_w^2,\ a.\ s. \tag{38}$$

Lemma 1: For the stochastic gradient algorithm in (25)-(27), we have

$$\sum_{i=1}^{L} h_i(z(k-1))C_i(q^{-1})\varsigma(k) = \beta(k) \tag{39}$$

Proof: The proof is given in Appendix C. ■

In addition to the results in Lemma 1, we shall need the following assumptions in order to obtain some properties of the parameter estimate $\widehat{\theta}(k)$.

Assumption 2 : For each i, $1 \le i \le L$, system $C_i(q^{-1})$ is input strictly passive (ISP) [10].

In (39), the signals $\varsigma(k)$ and $\beta(k)$ are related by the fuzzy polynomial $\sum_{i=1}^{L} h_i(z(k-1))C_i(q^{-1})$. As shall be shown in the next lemma, **Assumption 2** implies a passivity condition for that fuzzy polynomial.

Lemma 2: Consider the fuzzy system in (39). With **Assumption 2** that $C_i(q^{-1})$ is input strictly passive (ISP), we have

$$\sum_{j=1}^{k}\beta(j)\varsigma(j) - \epsilon\varsigma^2(j) \ge 0,\ for\ k \ge 1 \tag{40}$$

for some $\epsilon > 0$.

Proof: The proof is given in Appendix D. ■

Theorem 3: Under **Assumption 2**, for the stochastic gradient algorithm in (25)-(29), we have the parameter difference convergence

$$\lim_{N\to\infty}\sum_{k=1}^{N}\left\|\widehat{\theta}(k) - \widehat{\theta}(k-1)\right\|^2 < \infty,\ a.\ s. \tag{41}$$

and the normalized prediction error convergence

$$\lim_{N\to\infty}\sum_{k=1}^{N}\frac{[e(k) - w(k)]^2}{r(k-1)} < \infty,\ a.\ s. \tag{42}$$

Proof: With the help of previous lemmas, the results can be conducted along the same line made in Theorem 8.5.1 in [10]. Therefore the proof is omitted. ■

E. Adaptive Minimum Variance Control

To propose a direct adaptive fuzzy minimum variance controller, we shall first discuss the structure of the non-adaptive minimum variance controller by assuming that the system parameters are given. For the fuzzy stochastic system (7) having the optimal one-step ahead prediction form in (21), the minimum variance tracking control minimizing the cost function $J_1(k+1)$ in (11) is given by [19]

$$u(k) = \frac{1}{b_0(k)}\left\{-\sum_{i=1}^{L}h_i(z(k))\left[B_i(q^{-1}) - b_{i0}\right]u(k) \right. \\ \left. + \sum_{i=1}^{L}h_i(z(k))\left[C_i(q^{-1})y^*(k+1) - \alpha_i(q^{-1})y(k)\right]\right\} \tag{43}$$

where $b_0(k) = \sum_{i=1}^{L} h_i(z(k))b_{i0}$. The effect of the control law in (43) is to give

$$y^0(k+1|k) = y^*(k+1) = \phi^T(k)\theta_0, \tag{44}$$

i.e., the predicted output is forced to be equal to the desired output. Now suppose that the estimated parameters, $\widehat{\alpha}_{ij}(k), \hat{b}_{ij}(k)$, and $\hat{c}_{ij}(k)$ are obtained by using the stochastic gradient algorithm at time k. Accordingly, denote $\widehat{\alpha}_i(k, q^{-1})$, $\hat{B}_i(k, q^{-1})$, and $\hat{C}_i(k, q^{-1})$ be the estimates of the polynomials $\alpha_i(q^{-1})$, $B_i(q^{-1})$, and $C_i(q^{-1})$ at time index k, respectively. Also let $\widehat{b}_0(k) = \sum_{i=1}^{L} h_i(z(k))\hat{b}_{i0}(k)$. Based on the above estimated polynomials, the adaptive minimum variance control law is given by

$$u(k) = \frac{1}{\widehat{b}_0(k)}\sum_{i=1}^{L}h_i(z(k))\left\{-\left[\widehat{B}_i(k, q^{-1}) - \widehat{b}_{i0}(k)\right]u(k) \right. \\ \left. + \left[\widehat{C}_i(k, q^{-1})y^*(k+1) - \widehat{\alpha}_i(k, q^{-1})y(k)\right]\right\} \tag{45}$$

in which the control law is derived from the following equation

$$y^*(k+1) = \phi^T(k)\widehat{\theta}(k) \tag{46}$$

F. Analysis of Stability And Tracking Performance

In this section, stability and tracking performance of the proposed adaptive stochastic fuzzy control system will be discussed. As the output $y(k)$ is demanded to track arbitrary bounded reference signal $y^*(k)$, some minimum-phase-like property of the stochastic fuzzy system in (7) is required in order to ensure internal stability of the adaptive control system. Therefore, we make the following assumption.

Assumption 3: (i) Assume that these exists a positive number $b_{0,\min}$ such that $0 < b_{0,\min} \leq b_{0,i}$ and thus $0 < b_{0,\min} \leq |b_{\mathbf{0}}(k)|$. (ii) Let $\Xi_{\widetilde{B},i}$ be the companion matrix associated with the polynomial $\widetilde{B}_i(q^{-1})$ which is defined as $\widetilde{B}_i(q^{-1}) = \frac{B_i(q^{-1})}{b_{0,\min}}$. Assume that there exist $m \times m$ symmetric positive definite matrices $P_{\widetilde{B},i}$, for $1 \leq i \leq L$, of the form

$$P_{\widetilde{B},i} = \begin{bmatrix} P_{\widetilde{B},i}^{11} & 0_{(m-1)\times 1} \\ 0_{1\times(m-1)} & P_{\widetilde{B},i}^{22} \end{bmatrix} \tag{47}$$

such that the matrix inequalities

$$\begin{bmatrix} \lambda_{\widetilde{B}} P_{\widetilde{B},i} & \Xi_{\widetilde{B},i}^T P_{\widetilde{B},j} \\ P_{\widetilde{B},j}\Xi_{\widetilde{B},i} & P_{\widetilde{B},j} \end{bmatrix} > 0,\ 1 \leq i,\ j \leq L \tag{48}$$

hold for some $\lambda_{\widetilde{B}}$ with $0 < \lambda_{\widetilde{B}} < 1$.

Based on **Assumption 3,** we have the following results which will be used to prove stability of the adaptive control system.

Lemma 3: Under **Assumption 3**, for the stochastic fuzzy system in (7), we have, for $N \geq \overline{N}$,

$$\frac{1}{N}\sum_{k=1}^{N-1} \|u(k)\|^2 \leq \frac{K_6}{N}\sum_{k=1}^{N+1} \|y(k)\|^2 + K_7, \text{ a.s.} \tag{49}$$

where $0 < K_6 < \infty$, $0 < K_7 < \infty$, and $\overline{N}$ is a sufficient large number.

Proof: The proof is given in Appendix E. ■

Lemma 4: Under **Assumption 1-Assumption 3**, there exist finite positive constants K_8 to K_{11}, K_{a1} , and K_{a2} such that

$$\frac{1}{N}\sum_{k=1}^{N} y^2(k) \leq \frac{K_8}{N}\sum_{k=1}^{N} [e(k)-w(k)]^2 + K_9, a.s. \tag{50}$$

$$\frac{1}{N}\sum_{k=1}^{N} \overline{y}^2(k) \leq \frac{K_{10}}{N}\sum_{k=1}^{N} [e(k)-w(k)]^2 + K_{11}, a.s. \tag{51}$$

$$\frac{r(N-1)}{N} \leq \frac{K_{a2}}{N}\sum_{k=1}^{N} [e(k)-w(k)]^2 + K_{a1}, a.s. \tag{52}$$

for $N \geq \overline{N}$.

Proof: The proof is given in Appendix F. ■

Based on the property in (42) and the last lemma, we can attain of the following *stochastic key technical lemma.*

Lemma 5: With the property in (42), if there exist positive constants K_{a1}, K_{a2}, and $\overline{N}$ such that

$$\frac{1}{N}r(N-1) \leq K_{a1} + \frac{K_{a2}}{N}\sum_{k=1}^{N} [e(k)-w(k)]^2,\ a.\ s., \text{ for } N \geq \overline{N} \tag{53}$$

then we have

$$(i)\ \lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} [e(k)-w(k)]^2 = 0,\ a.\ s. \tag{54}$$

$$(ii)\ \limsup_{k\to\infty} \frac{r(N-1)}{N} < \infty, a.\ s. \tag{55}$$

$$(iii)\ \lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} E\left\{[y(k)-\widehat{y}(k)]^2 \mid F_{k-1}\right\} = \sigma_w^2,\ a.\ s. \tag{56}$$

If, in addition, condition (5) is strengthened to the following ergodic condition

$$E\left\{w^4(k) \mid F_{k-1}\right\} < \infty,\ a.\ s. \tag{57}$$

then the property in (56) can be also strengthened to

$$(iv)\ \lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} [y(k)-\widehat{y}(k)]^2 = \sigma_w^2,\ a.\ s. \tag{58}$$

Proof: The proof can be made along the same line as the proof of Lemma 8.5.3 in [10]. ■

With the above lemma, we have the following tracking performance and global stability results.

Theorem 4: For the stochastic fuzzy system in (7) with **Assumption 1-Assumption 3**, the adaptive minimum variance control algorithm is internally stable with tracking performance as

$$(i)\ \limsup_{k\to\infty} \frac{1}{N}\sum_{k=1}^{N} y^2(k) < \infty, \quad a.\ s. \tag{59}$$

$$(ii)\ \limsup_{k\to\infty} \frac{1}{N}\sum_{k=1}^{N} u^2(k) < \infty, \quad a.\ s. \tag{60}$$

$$(iii)\ \lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} E\left\{[y(k)-y^*(k)]^2 \mid F_{k-1}\right\} = \sigma_w^2 \quad a.\ s. \tag{61}$$

Furthermore, if (57) holds, then the result (61) is strengthened to

$$(iv)\ \lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} [y(k)-y^*(k)]^2 = \sigma_w^2 \quad a.\ s. \tag{62}$$

G. Simulation Study

In this section, a simulation example is given to verify the proposed adaptive minimum variance control algorithm.

Example 1: Consider the following stochastic fuzzy system:

$$\begin{aligned} &If\ z(k)\ is\ F_i \\ &then\ A_i(q^{-1})y(k+1) = B_i(q^{-1})u(k) + C_i(q^{-1})w(k+1) \\ &for\ i = 1, 2, \cdots 5 \end{aligned}$$

where

$$
\begin{aligned}
A_1(q^{-1}) &= 1 - 0.27q^{-1} + 0.011q^{-2} \\
A_2(q^{-1}) &= 1 - 0.33q^{-1} + 0.023q^{-2} \\
A_3(q^{-1}) &= 1 - 0.36q^{-1} + 0.0288q^{-2} \\
A_4(q^{-1}) &= 1 - 0.39q^{-1} + 0.035q^{-2} \\
A_5(q^{-1}) &= 1 - 0.44q^{-1} + 0.0468q^{-2} \\
B_1(q^{-1}) &= 1 - 0.2q^{-1},\ C_1(q^{-1}) = 1 - 0.135q^{-1} \\
B_2(q^{-1}) &= 1 - 0.3q^{-1},\ C_2(q^{-1}) = 1 - 0.165q^{-1} \\
B_3(q^{-1}) &= 1 - 0.4q^{-1},\ C_3(q^{-1}) = 1 - 0.18q^{-1} \\
B_4(q^{-1}) &= 1 - 0.5q^{-1},\ C_4(q^{-1}) = 1 - 0.195q^{-1} \\
B_5(q^{-1}) &= 1 - 0.6q^{-1},\ C_5(q^{-1}) = 1 - 0.22q^{-1}
\end{aligned}
$$

and $w(k)$ is a zero-mean Gaussian white noise with $\sigma_w^2 = 0.01$. The membership function for the fuzzy logic set F_i is given in Fig 1 and the premise variable $z(k)$ is chosen as $z(k) = y(k)$. We choose $y^*(k+1) = \sin(\frac{2\pi k}{100}) + 3\sin\left(\frac{6\pi k}{100}\right)$ as the reference signal. In Fig. 2, the adaptive minimum variance control $u(k)$ is shown in the upper trace, while the output $y(k)$ and the reference signal $y^*(k)$ are compared in the lower trace. Obviously, a usual transient phase of the adaptive control can be observed. Fig. 2 verifies that the internal stability and the tracking performance of the closed-loop system. The standard deviation of the tracking error during the steady state is 0.1058, which is very close to the standard deviation $\sigma_w = 0.1$ of $w(k)$ as guaranteed in Theorem 4.

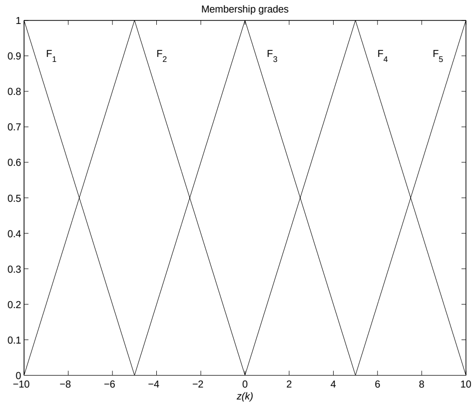

圖 1. Membership functions used in Example 1.

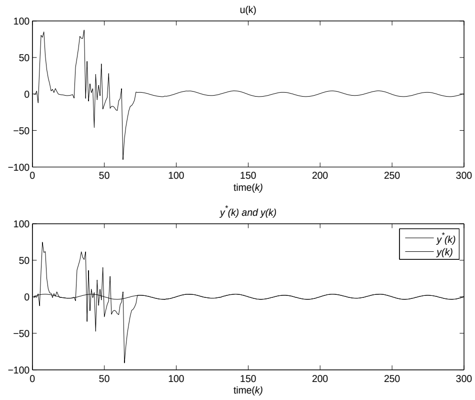

圖 2. Iutput $y(t)$ and its reference signal $y^*(t)$ in Example 1.

四、結論與討論

Adaptive minimum variance control for stochastic T-S fuzzy ARMAX model is addressed in this study. From the fuzzy ARMAX model, a fuzzy one-step ahead prediction model is first developed. A stochastic gradient algorithm is then proposed to identify the parameters of the related one-step-ahead predictor. Under the direct adaptive control scheme, the minimum variance control is applied to make the output track a desired reference signal. Stability and tracking performance of the adaptive stochastic fuzzy control system are rigorously derived and verified by simulation study.

參考文獻

[1] M. Sugeno and K. Tanaka, "Successive identification of Fuzzy model," *Fuzzy Set and Systems,* Vol. 28, pp. 156-33, 1988.

[2] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst. , Man, Cybern.*, Vol. 15, pp. 116-132, Jan/Feb, 1985.

[3] M. Sugeno and T. Yasukawa, "A fuzzy logic-based approach to qualitative modeling," *IEEE Trans. Fuzzy Systems*, Vol. 1, No. 1, pp.7-31, Feb. 1993.

[4] L. X. Wang, *Adaptive Fuzzy Systems and Control: Design and Stability Analysis*, Englewood Cliffs, NJ, Prentice-Hall, 1994.

[5] L. X. Wang, "Stable adaptive fuzzy control of nonlinear systems," *IEEE Trans. Fuzzy System*, Vol. 1, No 2, pp. 146-155, May. 1993.

[6] L. X. Wang, "Modeling and Control of hierachical systems with fuzzy systems," *Automatica*, Vol.33, No 6, pp. 1041-1053, June. 1997.

[7] T. K. Yin and C. S. George Lee, "Fuzzy model-reference adaptive control," *IEEE Trans. System, Man, Cybern.*, Vol.25, No. 12, pp.1606-1615, Dec. 1995.

[8] B. S. Chen, C. H. Lee, and Y. C. Chang, "H^∞ tracking design of uncertain nonlinear SISO systems: adaptive fuzzy approach," *IEEE Trans. Fuzzy Systems*, Vol.4, No. 1, pp. 32-43, Feb. 1996.

[9] D. Williamson, *Digital Control and Implementation*, Prentice Hall, Englewood Cliffs, N.J. 1991

[10] G. C. Goodwin and K.S. Sin, *Adaptive filtering prediction and control*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1984.

[11] K. Watanabe, "Stochastic fuzzy control. I. Theoretical derivation," *Proceedings of 1995 IEEE International Conference on Fuzzy Systems*, vol.2, pp. 547-554, March. 1995.

[12] K. Watanabe, K. Izumi and Fuha Han, "Stochastic fuzzy servo control using multiple linear dynamic models," *Proceedings of the Second International Conference on Knowledge-Based Intelligent Electronic Systems*, Vol.3, pp. 474-482, April. 1998.

[13] E. G. Laukonen, K. M. Passino, V. Krihnaswami, G.-C. Luh, and G. Rizzoni, "Fault detection and isolation for an experimental internal combustion engine via fuzzy identification," *IEEE Trans. Control System Technology*, Vol. 3, No. 3, pp. 347-355, Sep. 1995.

[14] J. Hu, K. Kumamaru, and K. Inoue, "A hybrid quasi-ARMAX modeling scheme for identification and control of nonlinear systems," *Proceedings of the 35th IEEE Conference on Decision and Control*, Vol. 2, pp. 1413 -1418, Dec. 1996.

[15] J. B. Waller, J. Hu, and K. Kirasawa, "Nonlinear model predictive control utilizing a neuro-fuzzy predictor," *2000 IEEE International Conference on Systems, Man, and Cybernetics*, Vol. 5, pp. 3459-3464, Oct. 2000.

[16] H.-T. Yang and C.-M. Huang, "A new short-term load forecasting approach using self-organizing fuzzy ARMAX models," *IEEE Transactions on Power Systems*, Vol. 13, No. 1, pp. 217-225, Feb. 1998.

[17] T. T. Ho, "Stochastic fuzzy direct adaptive control," *Proceedings of the Third IEEE Conference on Fuzzy Systems*, Vol.2, pp. 750-755, June 1994.

[18] A. N. Shiryayev, *Probability*, in Graduate Texts in Mathematics Series, Vol. 95, Springer-Verlag, New York, 1984.

[19] Bor-Sen Chen, Bore-Kuen Lee, and Ling-Bin Guo, "Optimal tracking design for stochastic fuzzy systems," *IEEE Transactions Fuzzy Systems*, , Vol. 11, No. 6, pp. 796-813, Dec. 2003.

Appendix

A. Proof of Theorem 1

Proof: First define a Lyapunov function as

$$V(x(k)) = x^T(k)P(k)x(k) \tag{A.1}$$

which is uniformly positive definite and

$$\lambda_P^{\min} \|x(k)\|^2 \le V(x(k)) \le \lambda_P^{\max} \|x(k)\|^2 \tag{A.2}$$

With the definition of $V(x(k))$, it follows that

$$V(x(k+1)) = x^T(k)\left[A^T(k)P(k+1)A(k)\right]x(k) \tag{A.3}$$

Note that the terms $x(k)$, $P(k)$, and $A(k)$ are all $\mathcal{F}_k-$measurable. Now applying the conditional mean operator $E\{\cdot \mid \mathcal{F}_k\}$ to the both sides of (A.3) and using (13), we have, almost surely,

$$\begin{array}{rl} & E\{V(x(k+1)) \mid \mathcal{F}_k\} \\ = & x^T(k)\left[A^T(k)E\{P(k+1)|\mathcal{F}_k\}A(k)\right]x(k) \\ \le & \lambda x^T(k)P(k)x(k) \\ = & \lambda V(x(k)) \end{array} \tag{A.4}$$

Note that as $E\left\{\|A(k)\|^2\right\}$ and $E\left\{\|P(k)\|^2\right\}$are uniformly bounded, $E\{V(x(k+1)) \mid \mathcal{F}_k\}$ and $E\{P(k+1)|\mathcal{F}_k\}$ are well defined. Apply the conditional expectation operator $E\{\cdot \mid \mathcal{F}_{k-1}\}$ again to the both sides of (A.4) and recall that the sequence of the $\sigma-$algebra $\mathcal{F}_k$ is increasing. With the smoothing properties [10] of conditional mean and inequality (A.4), it follows that almost surely

$$E\{V(x(k+1)) \mid \mathcal{F}_{k-1}\} \le \lambda^2 V(x(k-1))$$

Continuing this procedure by sequentially applying $E\{\cdot \mid \mathcal{F}_{k-2}\}$, $E\{\cdot \mid \mathcal{F}_{k-3}\}$, $\cdots$, $E\{\cdot \mid \mathcal{F}_{k_0}\}$, one can obtain almost surely

$$E\{V(x(k+1)) \mid \mathcal{F}_{k_0}\} \le \lambda^{k+1-k_0} V(x(k_0)) \tag{A.5}$$

Now we turn to prove the almost sure exponential stability (15). Clearly, it is trivial if $x(k_0) = 0$. Now assume that the initial condition $x(k_0)$ is nonzero. By Chebyshev's inequality [18], for any $\epsilon_k > 0$, we have

$$\begin{array}{rcl} \text{Prob}\left\{\frac{\|x(k)\|}{\|x(k_0)\|} > \epsilon_k\right\} & \le & E\left\{\frac{\|x(k)\|^2}{\|x(k_0)\|^2}\right\}/\epsilon_k^2 \\ & = & E\left\{\frac{1}{\|x(k_0)\|^2} \right. \\ & & \left. \times E\left\{\|x(k)\|^2 \mid \mathcal{F}_{k_0}\right\}\right\}/\epsilon_k^2 \end{array} \tag{A.6}$$

where Prob$\{\mathcal{A}\}$ is the probability measure of the event $\mathcal{A}$. With (A.2) and (A.5), one can get

$$E\left\{\|x(k)\|^2 \mid \mathcal{F}_{k_0}\right\} \le \frac{\lambda_P^{\max}}{\lambda_P^{\min}}\lambda^{k-k_0}\|x(k_0)\|^2, \ a.s.$$

With the last inequality, (A.6) can be reduced to

$$\text{Prob}\left\{\frac{\|x(k)\|}{\|x(k_0)\|} > \epsilon_k\right\} \le \frac{1}{\epsilon_k^2}\frac{\lambda_P^{\max}}{\lambda_P^{\min}}\lambda^{k-k_0} \tag{A.7}$$

Now choose the sequence ϵ_k as $\epsilon_k = \epsilon_0\lambda_1^{(k-k_0)/2}$ for any $\epsilon_0 > 0$ and $\lambda_1 > \lambda$. Then inequality (A.7) implies that

$$\begin{aligned} &\sum_{k=k_0}^{\infty} \text{Prob}\left\{\|x(k)\| > \epsilon_0\lambda_1^{(k-k_0)/2}\|x(k_0)\|\right\} \\ &\le \frac{1}{\epsilon_0^2}\frac{\lambda_P^{\max}}{\lambda_P^{\min}}\sum_{k=k_0}^{\infty}(\frac{\lambda}{\lambda_1})^{k-k_0} \end{aligned}$$

As $\frac{\lambda}{\lambda_1} < 1$, it follows that

$$\sum_{k=k_0}^{\infty} \text{Prob}\left\{\|x(k)\| > \epsilon_0\lambda_1^{(k-k_0)/2}\|x(k_0)\|\right\} < \infty$$

and consequentially, by the Borel-Cantelli Lemma [18], we obtain that

$$\text{Prob}\left\{\cup_{k\ge K_1}\left\{\|x(k)\| > \epsilon_0\lambda_1^{(k-k_0)/2}\|x(k_0)\|\right\}\right\} = 0$$

for some sufficiently large K_1, any $\epsilon_0 > 0$, and any $\lambda_1 > \lambda$. This means that for any sample path with bounded initial state $x(k_0)$, we have

$$\|x(k)\| \le c_1(\sqrt{\lambda})^{k-k_0}\|x(k_0)\|, \quad k \ge K_1, \ a.s.$$

for any initial condition $x(k_0)$, some positive bounded random variable c_1, and a sufficiently large integer K_1. This completes the proof. ■

B. Proof of Theorem 2

Proof: Suppose that $\|A(k)\| \le A_L$, $\|B(k)\| \le B_L$, $\|C(k)\| \le C_L$, and $\|D(k)\| \le D_L$ for all k. Using the definition of the transition matrix defined in (16), the response of the output $y_s(k)$ of the fuzzy system in (12) can be represented by

$$\begin{aligned} y_s(k) \ = \ & C(k)\Phi(k,0)x(0) + D(k)u_s(k) \\ & + C(k)\sum_{j=0}^{k-1}\Phi(k,j+1)B(j)u_s(j) \end{aligned}$$

Applying the results in Corollary 1, for $k \ge K_1$, we have

$$\begin{aligned} \|y_s(k)\| \le\ & C_L c_2\sqrt{\lambda}^k\|x(0)\| + D_L\|u_s(k)\| \\ & + C_L B_L\sum_{j=0}^{k-1}\|\Phi(k,j+1)\|\|u_s(j)\|, \ a.\ s. \end{aligned}$$

By the Cauchy-Schwartz inequality, the last inequality leads to

$$\begin{aligned} \|y_s(k)\|^2 \le\ & 3\{C_L^2c_2^2\lambda^k\|x(0)\|^2 + D_L^2\|u_s(k)\|^2 \\ & + C_L^2B_L^2\left[\sum_{j=0}^{k-1}\|\Phi(k,j+1)\|\|u_s(j)\|\right]^2\}, \ a.\ s. \\ & \le c_3\lambda^k + 3D_L^2\|u_s(k)\|^2 \\ & + 3C_L^2B_L^2\sum_{j=0}^{k-1}\|\Phi(k,j+1)\| \\ & \times\sum_{j=0}^{k-1}\|\Phi(k,j+1)\|\|u_s(j)\|^2, \ a.\ s. \end{aligned} \tag{B.1}$$

where c_3 is defined as $c_3 = 3C_L^2 c_2^2 \|x(0)\|^2$. Considering the change of index $i = k - j$, the first summation term in the last inequality can be rearranged as

$$\begin{aligned}\sum_{j=0}^{k-1} \|\Phi(k, j+1)\| &= \sum_{i=1}^{k} \|\Phi(k, k-i+1)\| \\ &= \sum_{i=1}^{K_1} \|\Phi(k, k-i+1)\| \\ &+ \sum_{i=K_1+1}^{k} \|\Phi(k, k-i+1)\|\end{aligned} \tag{B.2}$$

With the transition matrix defined in (16), it follows that $\|\Phi(k, k-i+1)\| \leq A_L^{i-1}$ for $i \leq K_1$. On the other hand, for $i > K_1$, inequality (17) ensures that $\|\Phi(k, k-i+1)\| \leq c_2 \sqrt{\lambda}^{i-1}$, $a.s.$ and thus

$$\begin{aligned}\lim_{k\to\infty} \sum_{j=0}^{k-1} \|\Phi(k, j+1)\| &\leq \sum_{i=1}^{K_1} A_L^{i-1} + c_2 \sum_{i=K_1+1}^{\infty} \sqrt{\lambda}^{i-1} \\ &= c_4 < \infty, \;\; a.\ s.\end{aligned} \tag{B.3}$$

where

$$c_4 = \frac{1 - A_L^{K_1}}{1 - A_L} + c_2 \frac{\sqrt{\lambda}^{K_1}}{1 - \sqrt{\lambda}}$$

Taking the summation operation $\frac{1}{N}\sum_{k=1}^{N}$ on both sides of (B.1) and using (B.3), one can get

$$\begin{aligned}\frac{1}{N}\sum_{k=1}^{N} \|y_s(k)\|^2 &\leq \frac{1}{N}\frac{c_3}{1-\lambda} + \frac{3D_L^2}{N}\sum_{k=1}^{N}\|u_s(k)\|^2 \\ &+ \frac{3C_L^2 B_L^2 c_4}{N} \\ &\times \sum_{k=1}^{N}\sum_{j=0}^{k-1} \|\Phi(k, j+1)\| \|u_s(j)\|^2, \quad a.\ s.\end{aligned} \tag{B.4}$$

in which the double summation term can be rearranged as follows

$$\begin{aligned}&\sum_{k=1}^{N}\sum_{j=0}^{k-1} \|\Phi(k, j+1)\| \|u_s(j)\|^2 \\ &= \sum_{j=0}^{N-1}\sum_{k=j+1}^{N} \|\Phi(k, j+1)\| \|u_s(j)\|^2\end{aligned}$$

With the same argument made from (B.2) to (B.3), it is easy to see that

$$\sum_{k=j+1}^{N} \|\Phi(k, j+1)\| \leq \sum_{k=j+1}^{\infty} \|\Phi(k, j+1)\| \leq c_4 < \infty, \quad a.\ s. \tag{B.5}$$

Therefore, following from (B.4) and (B.5), inequality (18) can be attained with

$$\begin{aligned}K_3 &= \tfrac{c_3}{1-\lambda} + 3C_L^2 B_L^2 c_4^2 \|u_s(0)\|^2 \\ &= 3C_L^2 \left(\tfrac{c_2^2}{1-\lambda}\|x(0)\|^2 + B_L^2 c_4^2 \|u_s(0)\|^2\right) \\ K_2 &= \max\left(3D_L^2,\; 3C_L^2 B_L^2 c_4^2\right)\end{aligned}$$

■

C. Proof of Lemma 1

Proof: Rewrite (7) to get

$$\begin{aligned}&\sum_{i=1}^{L} h_i(z(k-1)) A_i(q^{-1}) y(k) \\ &= \sum_{i=1}^{L} h_i(z(k-1)) \left[B_i(q^{-1}) u(k-1) + C_i(q^{-1}) w(k)\right]\end{aligned} \tag{C.1}$$

Substituting (20) into (C.1), we have

$$\begin{aligned}&\sum_{i=1}^{L} h_i(z(k-1))[C_i(q^{-1}) - q^{-1}\alpha_i(q^{-1})] y(k) \\ &= \sum_{i=1}^{L} h_i(z(k-1)) \big[B_i(q^{-1}) u(k-1) \\ &+ C_i(q^{-1}) w(k)\big]\end{aligned}$$

which leads to

$$\begin{aligned}&\sum_{i=1}^{L} h_i(z(k-1)) C_i(q^{-1})[y(k) - w(k)] \\ &= \sum_{i=1}^{L} h_i(z(k-1)) \big[q^{-1}\alpha_i(q^{-1}) y(k) \\ &+ B_i(q^{-1}) u(k-1)\big]\end{aligned}$$

From (32) and (33), we subtract $\sum_{i=1}^{L} h_i(z(k-1)) C_i(q^{-1})\overline{y}(k)$ from both sides of the above equation to get

$$\begin{aligned}&\sum_{i=1}^{L} h_i(z(k-1)) C_i(q^{-1})[y(k) - \overline{y}(k) - w(k)] \\ &= \sum_{i=1}^{L} h_i(z(k-1)) \big[q^{-1}\alpha_i(q^{-1}) y(k) \\ &- C_i(q^{-1})\overline{y}(k) + B_i(q^{-1}) u(k-1)\big]\end{aligned}$$

and thus

$$\begin{aligned}&\sum_{i=1}^{L} h_i(z(k-1)) C_i(q^{-1}) \varsigma(k) \\ &= \sum_{i=1}^{L} h_i(z(k-1)) \big[-(C_i(q^{-1}) - 1)\overline{y}(k) \\ &+ q^{-1}\alpha_i(q^{-1}) y(k) + B_i(q^{-1}) u(k-1) - \overline{y}(k)\big]\end{aligned}$$

Using (27), we can get the following equation

$$\begin{aligned}&\sum_{i=1}^{L} h_i(z(k-1)) C_i(q^{-1}) \varsigma(k) \\ &= \sum_{i=1}^{L} h_i(z(k-1)) \chi^T(k-1)\theta_{i0} - \overline{y}(k) \\ &= \phi^T(k-1)\theta_0 - \phi^T(k-1)\widehat{\theta}(k) = -\phi^T(k-1)\widetilde{\theta}(k) \\ &= \beta(k)\end{aligned}$$

This completes the proof. ■

D. Proof of Lemma 2

Proof: First define $\beta_i(k) = C_i(q^{-1})\varsigma(k)$ for $1 \le i \le L$. With the fuzzy system (39), we have

$$\beta(k) = \sum_{i=1}^{L} h_i(z(k-1))\beta_i(k) \tag{D.1}$$

As $C_i(q^{-1})$ is ISP [10], for any i, there is a positive number ϵ_i such that

$$\sum_{j=1}^{k} \varsigma(j)\beta_i(j) - \epsilon_i \varsigma^2(j) \ge 0 \tag{D.2}$$

Taking the operation $\sum_{i=1}^{L} h_i(z(k-1))$ on both side of (D.2) gives

$$\sum_{j=1}^{k} \left\{ \sum_{i=1}^{L} h_i(z(k-1)) \left[\varsigma(j)\beta_i(j) - \epsilon_i \varsigma^2(j)\right] \right\} \ge 0 \tag{D.3}$$

Using equation (D.1) and letting $\epsilon = \min_{1 \le i \le L} \epsilon_i$, we can see that inequality (D.3) implies the desired property in inequality (40). ■

E. Proof of Lemma 3

Before presenting the proof of Lemma 3, we shall need a lemma which is quoted from [19].

Lemma 6: Let P be a $m \times m$ symmetric positive definite matrix which is partitioned as

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}$$

where P_{11} and P_{22} are $(m-1)\times(m-1)$ and 1×1 matrices, respectively. Also let Γ be a matrix defined by

$$\Gamma = \begin{bmatrix} I_{m-1} & 0_{(m-1)\times 1} \\ 0_{1\times(m-1)} & r \end{bmatrix}, \; 0 < |r| \le 1$$

Then $\Gamma^T P \Gamma - P$ is negative semi-definite if and only if $P_{12} = 0_{(m-1)\times 1}$.

The proof of Lemma 3 is given in the following.

Proof: First define a function $V_1(k)$ as

$$V_1(k+1) = \sum_{i=1}^{L} h_i(z(k))\{A_i(q^{-1})y(k+1) - C_i(q^{-1})w(k+1)\} \tag{E.1}$$

so that from the stochastic fuzzy system in (7), we have

$$\begin{aligned}\sum_{k=1}^{N} h_i(z(k))b_{i0}u(k) = \sum_{k=1}^{N} &\left\{h_i(z(k))\left[b_{i0} - B_i(q^{-1})\right]u(k)\right. \\ &\left.+V_1(k+1)\right\}\end{aligned} \tag{E.2}$$

which can be expressed as

$$\begin{aligned} u(k) &= \frac{b_{0,\min}}{bo(k)} \sum_{i=1}^{L} \left\{ h_i(z(k)) \frac{\left[b_{i0} - B_i(q^{-1})\right]}{b_{0,\min}} u(k) \right. \\ &\left. + \frac{b_{0,\min}}{bo(k)} \frac{1}{b_{0,\min}} V_1(k+1) \right\} \\ &= -\widetilde{b}_0(k) \sum_{i=1}^{L} \{ h_i(z(k)) \left[\sum_{j=1}^{m} \frac{b_{ij}}{b_{0,\min}} u(k-j) \right] \\ &+ \widetilde{b}_0(k) V_2(k+1) \} \end{aligned} \tag{E.3}$$

where

$$\begin{array}{lcl} \widetilde{b}_0(k) & = & b_{0,\min}/bo(k) \\ V_2(k+1) & = & \frac{1}{b_{0,\min}} V_1(k+1) \end{array} \tag{E.4}$$

Note that $|b_{\mathbf{0}}(k)| \ge b_{0,\min}$ and thus $\widetilde{b}_0(k)$ is well defined with $0 < b_{0,\min} \le \left|\widetilde{b}_0(k)\right| \le 1$. By constructing a state vector $x_u(k)$ as

$$x_u(k) = \left[u(k-m) \; u(k-m+1) \; \cdots \; u(k-1)\right]^T,$$

equation (E.3) can be transformed into a state-space form as

$$x_u(k+1) = A'_{\widetilde{B}}(k)x_u(k) + V_u(k+1) \tag{E.5}$$

where

$$A_{\widetilde{B}}(k) = \sum_{i=1}^{L} h_i(z(k))\Xi_{\widetilde{B},\,i}$$
$$A'_{\widetilde{B}}(k) = \Gamma(k)A_{\widetilde{B}}(k)$$

$$\Gamma(k) = \begin{bmatrix} I_{m-1} & 0_{(m-1)\times 1} \\ 0_{1\times(m-1)} & \widetilde{b}_0(k) \end{bmatrix}$$
$$V_u(k+1) = \begin{bmatrix} 0_{1\times(m-1)} & \widetilde{b}_0(k)V_2(k+1) \end{bmatrix}^T$$

For the system (E.5), consider a Lyapunov function $V(x(k)) = x^T(k)P_{\widetilde{B}}(k)x(k)$ where $P(k) = \sum_{i=1}^{L} h_i(z(k))P_{\widetilde{B},i}$. With the structure defined in (47), by applying Lemma 6, it follows that

$$\begin{aligned} &\Gamma^T(k)P(k+1)\Gamma(k) \\ &= \sum_{i=1}^{L} h_i(z(k+1))\Gamma^T(k)P_{\widetilde{B},i}\Gamma(k) \\ &\le P(k+1) \end{aligned}$$

On the other hand, under **Assumption 3**, the matrix condition (48) implies

$$\lambda_{\widetilde{B}} P(k) - A^T_{\widetilde{B}}(k) E\left\{P(k+1)|\mathcal{F}_k\right\} A_{\widetilde{B}}(k) > 0, \; \forall k$$

for some $\lambda_{\widetilde{B}}$ with $0 < \lambda_{\widetilde{B}} < 1$. Therefore, as $\Gamma(k)$ is $\mathcal{F}_k-$measurable, we have

$$\begin{aligned} &A'^T_{\widetilde{B}}(k) E\left\{P(k+1)|\mathcal{F}_k\right\} A'_{\widetilde{B}}(k) \\ &= A^T_{\widetilde{B}}(k) E\left\{\Gamma^T(k)P(k+1)\Gamma(k)|\mathcal{F}_k\right\} A_{\widetilde{B}}(k) \\ &\le A^T_{\widetilde{B}}(k) E\left\{P(k+1)|\mathcal{F}_k\right\} A_{\widetilde{B}}(k) \\ &< \lambda_{\widetilde{B}} P(k) \end{aligned} \tag{E.6}$$

which implies that for the system in (E.5), it follows from Theorem 2 that

$$\begin{aligned}&\frac{1}{N}\sum_{k=1}^{N}\|x_u(k)\|^2\\&\le\frac{K_2}{N}\sum_{k=1}^{N}\|V_u(k+1)\|^2+\frac{K_3}{N},\quad a.\ s.,\ \text{for } N\ge K_1\end{aligned}\tag{E.7}$$

For the left hand side of (E.7), by the definition of the vector $x_u(k)$, it follows

$$\frac{1}{N}\sum_{k=1}^{N-1}\|u(k)\|^2\le\frac{1}{N}\sum_{k=1}^{N}\|x_u(k)\|^2\tag{E.8}$$

For the left hand side of (E.7), by the definition of $V_u(k+1)$, we have

$$\begin{aligned}&\|V_u(k+1)\|^2\\&=\left\{\frac{\widetilde{b}_0(k)}{b_{0,\min}}\sum_{i=1}^{L}h_i(z(k))\left(\theta_{A_i}^T\phi_y(k+1)-\theta_{C_i}^T\phi_w(k+1)\right)\right\}^2\end{aligned}$$

where

$$\begin{aligned}\theta_{A_i}&=\left[\begin{array}{cccc}1 & a_{i1} & \cdots & a_{in]}\end{array}\right]^T\\\theta_{C_i}&=\left[\begin{array}{cccc}1 & c_{i1} & \cdots & c_{il]}\end{array}\right]^T\\\phi_y(k+1)&=\left[\begin{array}{cccc}y(k+1) & y(k) & \cdots & y(k+1-n)\end{array}\right]^T\\\phi_w(k+1)&=\left[\begin{array}{cccc}w(k+1) & w(k) & \cdots & w(k+1-l)\end{array}\right]^T\end{aligned}$$

Let $C_{AC}=\max\left\{\max_{1\le i\le L}\|\theta_{A_i}\|,\max_{1\le i\le L}\|\theta_{C_i}\|\right\}$. As $0<b_{0,\min}\le\left|\widetilde{b}_0(k)\right|\le 1$, it follows

$$\|V_u(k+1)\|^2\le\frac{2C_{AC}^2}{b_{0,\min}^2}(\|\phi_y(k+1)\|^2+\|\phi_w(k+1)\|^2)\tag{E.9}$$

Meanwhile, similar to (E.8), one can obtain

$$\frac{1}{N}\sum_{k=1}^{N}\|\phi_y(k+1)\|^2\le(n+1)\frac{1}{N}\sum_{k=1}^{N+1}y^2(k)\tag{E.10}$$

$$\frac{1}{N}\sum_{k=1}^{N}\|\phi_w(k+1)\|^2\le(l+1)\frac{1}{N}\sum_{k=1}^{N+1}w^2(k)\tag{E.11}$$

Therefore, by (E.7)-(E.11) and (5), we have, for $N\ge\overline{N}$,

$$\frac{1}{N}\sum_{k=1}^{N-1}\|u(k)\|^2\le K_6\times\frac{1}{N}\sum_{k=1}^{N+1}\|y(k)\|^2+K_7$$

where $\overline{N}=\max\{N_w,K_1\}$

$$K_6=\frac{2C_{AC}^2K_2(n+1)}{b_{0,\min}^2},\ K_7=\frac{2C_{AC}^2K_2K_w(l+1)}{b_{0,\min}^2}$$

This completes the proof. ■

F. Proof of Lemma 4

Proof: (i) The proof can be referred to part (iii) of Lemma 11.3.1 in [10].

(ii) The proof can be referred to part (iv) of Lemma 11.3.1 in [10].

(iii) In (29) with $k=N$, we have

$$\begin{aligned}&r(N-1)\\&=r(0)+\sum_{k=1}^{N-1}\phi^T(N-1)\phi(N-1)&\text{(F.1)}\\&=r(0)+\sum_{k=1}^{N-1}\sum_{i=1}^{L}h_i^2(z(k))\chi^T(k)\chi(k)&\text{(F.2)}\\&\le r(0)+\sum_{k=1}^{N-1}\sum_{i=1}^{L}h_i(z(k))\chi^T(k)\chi(k)&\text{(F.3)}\\&=r(0)+\sum_{k=1}^{N-1}\chi^T(k)\chi(k)&\text{(F.4)}\end{aligned}$$

By the definition of $\chi(k)$ in (27), it follows from (49),(50), and (51) that, for $N\ge\overline{N}$,

$$\frac{1}{N}r(N-1)\le\frac{K_{a2}}{N}\sum_{k=1}^{N-1}[e(k)-w(k)]^2+K_{a1}$$

for some positive numbers K_{a2} and K_{a1}. This completes the proof. ■

行政院國家科學委員會補助國內專家學者出席國際學術會議報告

97 年 07 月 21 日

附件三

報告人姓名	李柏坤	服務機構及職稱	中華大學電機系 教授
時間 會議 地點	97 年 7 月 12 日至 97 年 7 月 15 日 中國昆明	本會核定補助文號	國科會計畫 96-2221-E-216-036 中核定出席國際會議經費四萬元
會議名稱	（中文）2008 年機器學習與人工頭腦學國際研討會 (ICMLC 2008) （英文）2008 International Conference on Machine Learning and Cybernetics		
發表論文題目	1.（中文）隨機模糊 T-S ARMAX 模式之適應最小變異量控制 （英文）Adaptive Minimum Variance Control for Stochastic Fuzzy T-S ARMAX Model 2.（中文）具有狀態相關雜訊之隨機模糊 T-S 模式的 H∞輸出回饋控制 （英文）H∞ Output Feedback Control of Stochastic T-S Fuzzy Model with State-Dependent Noise		

一、參加會議經過:

此次2008年機器學習與人工頭腦學國際研討會(2008 International Conference on Machine Learning and Cybernetics ，ICMLC 2008)，由河北大學、IEEE SMC (System, Man, and Cybernetics) 協會、香港機器學習與控制研究所（MLCRI）等單位聯合主辦，於97年7月12日到97年7月15日，在中國昆明市Grand Park Hotel舉行。台灣科技大學校長陳希舜教授為 Honorary Conference Chairs 之一，台灣的學者參與此研討會非常踴躍。

此次研討會所有論文都列入IEEE Explorer之資料庫，都屬於EI Index。研討會之網路首頁為http://www.icmlc.com/，整個研討會包含四個Tutorials：

[1] Application of Neural Network and Cerebellar Model Articulation Controller in Control Problem, Speaker: Prof. Chih-Min Lin (元智大學電機系教授)

[2] Principles of Stochastic Discrimination and Ensemble Learning, Speaker: Prof. Tin Kam Ho

[3] Linguistic models: from data to granular architectures, Speaker: Prof. Witold Pedrycz

[4] Intelligence Pattern Recognition and Applications to Biometrics in an Interactive Environment, Speaker: Prof. Patrick Wang

另外有兩個Plenary Talk：

[1] Multimedia Information Security: An Overview of Research and Challenges, Speaker: Prof. Philip Chen

[2] - Alan Turing, spam e-mail, pattern recognition: an intriguing triangle, Speaker: Prof. Fabio Roli

此次研討會之主題包含：

1. Adaptive systems
2. Neural net and support vector machine
3. Business intelligence
4. Hybrid and nonlinear system
5. Biometrics
6. Fuzzy set theory, fuzzy control and system
7. Bioinformatics
8. Knowledge management
9. Data and web mining
10. Information retrieval
11. Intelligent agent,
12. Intelligent and knowledge based system
13. Financial engineering
14. Rough and fuzzy rough set
15. Inductive learning
16. Networking and information security
17. Geoinformatics
18. Evolutionary computation。

19. Pattern Recognition
20. Ensemble method
21. Logistics
22. Information fusion
23. Intelligent control
24. Visual information processing
25. Media computing
26. Computational life science

議程第一天安排了四個Tutorials，第二天上午有兩個Plenary talk，有不同小節之會議。

二、與會心得

(1) 從此次研討會所安排之主題來看，比較偏向人工智能於資訊工程之研究，各國有關人工智慧理論都有顯著的研究成果，幾個比較新的主題如 Media computing、Bioinformatics、Computational life science、Business intelligence，非常值得國內學界注意其發展。

(2) 除了認識許多中國之學者外，也認識了很多來自全世界各地的菁英學者，對於將來推動國際學術交流，有相當大的幫助。

(3) 大陸在人工智能領域之研究成果亦有長足之進步，在 IEEE SMC Society 之影響力也已超過台灣相關學界，國內應該即起直追。

三、考察參觀活動(無是項活動者省略)

主辦單位無舉辦任何考察參觀活動。

四、建議

(1) 台灣應該多爭取舉辦國際研討會，使得全世界各地的菁英學者，能夠共聚一堂。我覺得國內也可以由幾個學會或中國工程師協會，舉辦國際聯合研討會。

(2) 大陸學術界的國際化，已經逐步生根，同時以此為基礎邁向國際之競爭，台灣學術界的自由化與國際化還有待大家的努力。

五、攜回資料名稱及內容

(1) 完整論文光碟片。

(2) 論文摘要紙本以及部份之論文集。

六、其他

無。