

Single Choice (1~5, 3% each)

1. Find the domain of the function

$$f(x) = 3 + \sin x .$$

Select the correct answer.

- (A). $(-\infty, \infty)$ (B). $(2, \infty)$ (C). $(2, 4)$ (D). $[2, 4]$ (E). $[2, \infty)$

2. Find the range of the function

$$f(x) = 3 + \sin x .$$

Select the correct answer.

- (A). $(-\infty, \infty)$ (B). $(2, \infty)$ (C). $(2, 4)$ (D). $[2, 4]$ (E). $[2, \infty)$

3. Which of the following functions is an increasing function on \mathbb{R} .

Select the correct answer.

- (A). $\sin x$ (B). $|x|$ (C). x^2 (D). e^{-x} (E). none of these.

4. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = ?$

Select the correct answer.

- (A). 0 (B). 1 (C). -1 (D). ∞ (E). $-\infty$

5. Let $P(x)$ and $Q(x)$ be polynomials. Find

$$\lim_{x \rightarrow \infty} \frac{P(x)}{Q(x)}$$

if the degree of $P(x)$ is 5 and the degree of $Q(x)$ is 9.

Select the correct answer.

- (A). -4 (B). 9 (C). 4 (D). 0 (E). 5

Single Choice (6~15, 5% each)

6. Find f' in terms of g' , where $f(x) = x^2 g(x)$.

Select the correct answer.

(A). $f'(x) = 2xg(x) + x^2 g'(x)$

(B). $f'(x) = 2x + g'(x)$

(C). $f'(x) = x^2 g(x) + 2x^2 g'(x)$

(D). $f'(x) = 2xg'(x)$

(E). none of these.

7. Evaluate the limit: $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\sin \frac{1}{n} + \sin \frac{2}{n} + \cdots + \sin \frac{n-1}{n} \right]$.

- (A). 1 (B). $1 - \sin 1$ (C). $1 - \cos 1$ (D). $\sin 1$ (E). 0

8. Identify the planes that are perpendicular.

Select the correct answer.

- (A). $x + 10y - z = 6$, $-9x - y - 19z = 2$ (B). $x = 5x + 3y$, $-10x - 6y + 2z = -1$
(C). $8x + 5y = -3$, $9y + 6z = -1$ (D). $8x + 5y = -3$, $y + 6z = -1$
(E). $7x + 4y = -3$, $9y + 6z = 1$

9. Find the point at which the line given by the parametric equations below intersects the plane.

Plane : $2x + 4y - 3z = -48$; line: $x = 10 + 7t$, $y = -10$, $z = 7t$

Select the correct answer.

- (A). $(-38, -10, 28)$ (B). $(43, -10, -18)$ (C). $(38, -10, 28)$
(D). $(-43, -10, 18)$ (E). $(38, -10, -28)$

10. Evaluate the integral.

$$\int \frac{x^2}{x+2} dx$$

Select the correct answer.

- (A). $\frac{x^2}{2} - 2x + 4 \ln(x+2) + C$ (B). $\frac{x^2}{2} + 2x + 4 \ln(x-2) + C$
(C). $\frac{x^2}{2} + 2x + 4 \ln|x+2| + C$ (D). $\frac{x^2}{2} - 2x + 4 \ln|x+2| + C$
(E). $\frac{x^2}{2} - 2x + 2 \ln|x+2| + C$

11. Evaluate the indefinite integral.

$$\int x \cos 7x dx$$

Select the correct answer.

- (A). $\frac{1}{49} \sin 7x + \frac{x}{7} \cos 7x + C$ (B). $\frac{1}{7} \cos 7x + \frac{x}{7} \sin 7x + C$
(C). $\frac{x}{49} \cos 7x + \frac{x}{7} \sin 7x + C$ (D). $\frac{1}{49} \cos 7x + \frac{x}{7} \sin 7x + C$
(E). none of these

12. Evaluate the integral using the indicated trigonometric substitution.

$$\int \frac{x^3}{\sqrt{x^2 + 25}} dx; \quad x = 5 \tan \theta$$

Select the correct answer.

(A). $-\frac{2}{3}(x^2 + 25)^{3/2} + x^2\sqrt{x^2 + 25} + C$ (B). $(x^2 + 25)^{3/2} - 5\sqrt{x^2 + 25} + C$

(C). $\frac{1}{3}(x^2 + 25)^{3/2} - \sqrt{x^2 + 25} + C$ (D). $(x^2 + 25)^{3/2} - \sqrt{x^2 + 25} + C$

(E). $\frac{3}{2}(x + 25)^{3/2} - 25\sqrt{x + 25} + C$

13. Differentiate the function.

$$y = \ln(x^3 \sin^2 x)$$

Select the correct answer.

(A). $y' = \frac{3 \sin x + x \cos x}{x^3 \sin^2 x}$ (B). $y' = \frac{3 \sin x - 2x}{x \sin x}$ (C). $y' = \frac{3 \sin x + 2x \cos x}{x \sin x}$

(D). $y' = \frac{3 \cos x + 2x \sin x}{x \cos x}$ (E). $y' = \frac{6x^2 + \cos x \sin x}{\cos x}$

14. Find the limit.

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin 8x}$$

Select the correct answer.

(A). 0 (B). ∞ (C). $-\infty$ (D). -3 (E). $\frac{1}{8}$

15. Use the Chain Rule to find $\frac{\partial u}{\partial p}$.

$$u = \frac{x + y}{y + z}$$

$$x = p + 6r + 5t, \quad y = p - 6r + 5t, \quad z = p + 6r - 5t$$

Select the correct answer.

(A). $\frac{\partial u}{\partial p} = \frac{5t}{p}$ (B). $\frac{\partial u}{\partial p} = -\frac{20t}{p^3}$ (C). $\frac{\partial u}{\partial p} = -\frac{5t}{p^2}$

(D). $\frac{\partial u}{\partial p} = -\frac{25t}{p^2}$ (E). $\frac{\partial u}{\partial p} = \frac{25t}{p^2} + t$

Multiple Choice (16~20, 7% each) *每題有 5 選項，全對得 7 分，對 4 錯 1 得 4 分，對 3 錯 2 得 1 分，其餘情形得 0 分。

16. Let $f(x) = 2x^3 - 3x + 6$.

Select the correct answers.

- (A). $f(x)$ is continuous on \mathbb{R} .
(B). $f(x)$ is differentiable on \mathbb{R} .
(C). $f(x)$ is integrable from a to b , for any a, b on \mathbb{R} .
(D). $f(x)$ has real root(s) on the interval $(-2, -1)$.
(E). $f(x) \geq 1000$ for all x on \mathbb{R} .

17. If $f(0) = 4$, $f'(0) = 2$, $g(0) = 3$, $g'(0) = -5$, select the correct answers.

- (A). $(f + g)'(0) = -3$ (B). $(f \cdot g)(0) = 12$ (C). $(f \cdot g)'(0) = 14$
(D). $(\frac{f}{g})(0) = 1$ (E). $(\frac{f}{g})'(0) = 26$

18. Let $g(x) = \sin(x^2)$ and $f(x) = \sin^2 x$.

Select the correct answers.

- (A). $g'(x) = 2x \cos(x^2)$
(B). $g'(x) = 2x \sin(x^2)$
(C). $f'(x) = 2x \sin(x^2)$
(D). $f'(x) = 2(\sin x)(\cos x)$
(E). $f'(x) = \sin(2x)$

19. $f(x, y) = 4x^2 - 9xy + 2y^2$

Select the correct answers .

- (A). $f_x = 8x - 9y$ (B). $f_y = -9y + 4y$ (C). $f_{xy} = 8x - 9$
(D). $f_{yy} = 4$ (E). $f_{xx}|_{x=3} = 8$

20. Select the correct answers.

- (A). $\int_0^{\infty} e^{-2x} dx = -\frac{1}{2}$ (B). $\int_0^{\infty} e^{-2x} dx = \frac{1}{2}$
(C). $\int_{-\pi/2}^{\pi/2} \sin^3 \theta \cos^2 \theta d\theta = \frac{2}{15}$ (D). $\int_2^4 \int_0^3 (3 + 2xy) dx dy = 72$
(E). $\int_{-\infty}^{\infty} \frac{dx}{4x^2 + 4x + 5} = 0$