計畫編號: NSC92-2218-E-216-003- 01 93 07 31

計畫主持人: 吳玫瑩

報告類型: 精簡報告

。
在前書 : 本計畫可公開查

行政院國家科學委員會專題研究計畫 成果報告

93 9 13

Integrated Supply Chain Inventory Model for Deteriorating Item with Random Demand

主持人**:** 吳玫瑩 中華大學資訊管理系

計畫編號**: NSC 92-2218-E-216-003-**

執行期限**: 92** 年 **11** 月 **1** 日 至 **93** 年 **7** 月 **31** 日

analysis of variance is performed to illustrate how the key factors: service level, standard deviation of demand, lead time, and deterioration rate affect the optimal solution. This study could help a supply chain purchasing manager at that situation decide on how much to order as well as obtain the minimum joint total cost.

Keywords: Supply chain management, Random demand, Deteriorating items, Integrated policy

1. Introduction

According to studies of Tersine (1994), when demand was treated as continuous, the most frequently used distribution was the normal distribution. Liao and Shyu (1991) first presented a probabilistic inventory model in which the demand during lead time followed a normal distribution. Ben-Daya and Raouf (1994) extended Liao and Shyu's model and considered both order quantity and lead time as decision variables to minimize the sum of the total cost, where the shortages were neglected. Later, Ouyang et al. (1996) extended Ben-Daya and Raouf's model to consider the issue of shortages and allowed backorders or lost-sales. Moon and Choi (1998) improved Ouyang et al.'s (1996) model by simultaneously optimizing both order quantity and reorder point. Later, Ouyang et al. (2002) extended Moon and Choi's model to take quality-related cost into account in

Abstract

This study develops a random demand inventory model for deteriorating items in an integrated supply chain with three parties: the buyer, the producer and the supplier. A mathematical model is derived for minimizing the joint total cost of three parties with the optimal lot size and the number of deliveries. A numerical example shows that the integrated policy results in an impressive cost-reduction compared with any independent decision. Finally, an

 \mathcal{A}_i , et al. (\mathcal{A}_i) and \mathcal{A}_i (\mathcal{A}_i) and \mathcal{A}_i (\mathcal{A}_i) and \mathcal{A}_i

析探討一些關鍵因子如: 服務水準、

關鍵詞**:** 供應鏈管理,隨機需求,損

determining the optimal ordering policy.

In a highly competitive market, the minimum joint total cost could be achieved if all the partners are willing to co-operate. In some of the literature, models have been established for optimizing supply chain operations by minimizing the joint total cost. Kim and Ha (1997) developed an integrated inventory model to derive the minimal joint total cost between the buyer and the vendor where the deterioration issue was neglected. In real life situation, the inventory models are considered in which inventory is depleted not only by demand but also by decay. Thus studying the deteriorating inventory is becoming very important. In the literature, Ghare and Schrader (1963) were the first researchers to consider exponentially decaying inventory for a constant demand. Yang and Wee (2000) extended the Kim and Ha's (1997) inventory model to consider deterioration and the integration of vendor and buyer. Later, Rau et al. (2003) and Wu et al. (2003) extended the integrated model to consider multi-echelon inventory system for deteriorating items with different demand assumptions.

From the above literature review, we find that this study might be the first simultaneously to consider the case of random demand with a normal distribution and integrated multi-echelon inventory model for deteriorating items. The result in this study has shown that the integrated viewpoint is more economical compared with an independent viewpoint.

2. Assumption and notations

The mathematical model developed in this study is based on the following assumptions:

- (1) The demand rate is random with a normal distribution.
- (2) The production rate is deterministic and constant.
- (3) The deterioration rate is deterministic and constant.
- (4) The planning period is known.
- (5) The lead time is allowed.
- (6) Single buyer, producer and supplier with one type of item are considered.
- (7) Multiple lot-size deliveries per order are considered.
- (8) The delivery quantity is constant for each delivery.

In order to establish the inventory model, the following notations are used:

3. The inventory model development

The supply chain system under this study is illustrated in Fig.1, and the inventory level for deteriorating item for each party in the supply chain is depicted in Fig.2. In order to fulfill each demand, the supplier delivers raw materials to the producer's warehouse at a fixed period; the producer withdraws raw materials from his own warehouse and processes them into finished goods, then delivers finished goods to the buyer at a replenishment cycle.

3.1. The finished goods inventory model for the buyer

In this study, the demand is treated as a normal distribution with mean *D* and standard deviation σ . When a deteriorating item with an exponential distribution for its deterioration is considered, the lot size of q_B is obtained as follows:

$$
q_{B} = \frac{D}{\theta_{B}} \left(e^{\theta_{B}(t+LT)} - 1\right) + \frac{Z\sigma\sqrt{(t+LT)}}{e^{-\theta_{B}(t+LT)}} \tag{1}
$$

Under a random demand environment, when the lot size of q_B is ordered, a firm might faces overstock or stockout. Then, the expected overstock and stockout cost can be derived as follows:

$$
\begin{split} \textit{Expected \: overstock = } q_s F_s \bigg(\frac{q_s - x}{\sigma \sqrt{(t + LT)}} \bigg) - \bigg(x F_s \bigg(\frac{q_s - x}{\sigma \sqrt{(t + LT)}} \bigg) - \sigma \sqrt{(t + LT)} f_s \bigg(\frac{q_s - x}{\sigma \sqrt{(t + LT)}} \bigg) \bigg) \\ & = (q_s - x) F_s \bigg(\frac{q_s - x}{\sigma \sqrt{(t + LT)}} \bigg) + \sigma \sqrt{(t + LT)} f_s \bigg(\frac{q_s - x}{\sigma \sqrt{(t + LT)}} \bigg) \end{split} \tag{2}
$$
\n
$$
\textit{Expected \: stockout} = (x - q_s) + q_s F_s \bigg(\frac{q_s - x}{\sigma \sqrt{(t + LT)}} \bigg) - \bigg(\frac{x F_s \bigg(\frac{q_s - x}{\sigma \sqrt{(t + LT)}} \bigg)} - \sigma \sqrt{(t + LT)} f_s \bigg(\frac{q_s - x}{\sigma \sqrt{(t + LT)}} \bigg) \bigg) \\ & = (x - q_s) \bigg(1 - F_s \bigg(\frac{q_s - x}{\sigma \sqrt{(t + LT)}} \bigg) \bigg) + \sigma \sqrt{(t + LT)} f_s \bigg(\frac{q_s - x}{\sigma \sqrt{(t + LT)}} \bigg) \bigg) \tag{3}
$$

Thus, the buyer's total cost of finished goods per period *T* can be expressed as the sum of the order cost, receiving cost, holding cost, deteriorating cost, overstock cost and stockout cost, that is:

Where $\bigg)$ $\left(\frac{\min(q_B,q_P)-D(t+LT)-ss}{\theta_B}\right)$ $\int min(q_B, q_P) - D(t + LT)$ *B* $\frac{q_B, q_P - D(t + LT) - ss}{\theta_p}$ $\min(q_B, q_P) - D(t + LT) - ss$ is the total

holding quantity of finished goods during period (*t+LT*), and $(\min(q_B, q_P) - D(t + LT) - ss)$ is the total deteriorating quantity of finished goods during period (*t+LT*).

3.2. The inventory model for the producer

3.2.1. The producer's finished goods inventory model

When $t_p > t$, the producer is unable to satisfy the buyer's demand during period *t*, and results in a shortage. Hence, *qP* can be written as the following relationship:

 $\overline{\mathcal{L}}$ ļ ſ $-e^{-\theta_P t}$), if $t_p >$ ≤ $=\begin{cases} P & \text{if } t_p > t, \text{ then } \text{stockout } \text{for } t \text{ he } \text{ producer} \end{cases}$ q_R *if* $t_p \leq t$, then no stockout for the producer $q_p = \begin{cases} P & \text{if } p \leq r \end{cases}$ *P B P P* $\left[P^{\text{p}} \right] = \frac{1}{2} (1 - e^{-\theta_p t})$, if $t_p > t$, $\int f \, t_p \leq t$, $\frac{1}{\theta_n}(1-e^{-\theta_n})$ (5)

The total cost of finished goods for the producer per period *T* can be expressed as the sum of the setup cost, delivery cost, holding cost, deteriorating cost, and stockout cost, is:

$$
TC_{PP} = \left\{ \frac{S_P * n}{T} \right\} + \left\{ \frac{F_P * n}{T} \right\} + \left\{ \frac{\left(\frac{P * \min(t, t_P) - \min(q_B, q_P)}{\theta_P} \right) * H_P * n}{T} \right\} + \left\{ \frac{\left(P * \min(t, t_P) - \min(q_B, q_P) \right) * P_P * n}{T} \right\} + \left\{ \frac{\max(q_B - q_P, 0) * \beta_P * n}{T} \right\} \tag{6}
$$

Where $\left(\frac{P*\min(t,t_P)-\min(q_B,q_P)}{q}\right)$ J $\left(\frac{P*\min(t,t_P) - \min(q_B,q_P)}{q}\right)$ is the total l *P* θ holding quantity of finished goods

during period *t*, and $(P * min(t, t_P) - min(q_B, q_P))$ is the total deteriorating quantity of finished goods during period *t*.

3.2.2. The producer's warehouse inventory model

The total cost for the producer's warehouse of raw materials per period *T* can be expressed as the sum of the receiving cost, holding cost, and deteriorating cost, i.e.

$$
r_{C_{PW}} = \left\{\frac{F_{PW} * n}{T}\right\} + \left\{\frac{\left(\frac{q_{PW} - P * \min(t, t_P)}{\theta_{PW}}\right) * H_{PW} * n}{T}\right\} + \left\{\frac{\left(\frac{q_{PW} - P * \min(t, t_P)}{\theta_{PW}}\right) * P_{PW} * n}{T}\right\}
$$
\n(7)

holding quantity of raw materials during period *t*, and $(q_{pw} - P * \min(t, t_p))$ is the total deteriorating quantity of raw materials during period *t*.

Then, the producer's total cost,
$$
TC_P
$$
 is:
\n
$$
TC_P = TC_{PP} + TC_{PW}
$$
\n(8)

3.3. The inventory model for the supplier

Assume supplier's opening inventory to be Q_{PW} , and the ending inventory after period t to be q_{PW} . Then, total cost of raw materials for the supplier per period *T* can be expressed as the sum of the order cost, delivery cost, holding cost, and deteriorating cost, i.e.

$$
TC_s = \left\{\frac{S}{T}\right\} + \left\{\frac{F_s * n}{T}\right\} + \left\{\frac{\frac{Q_{\text{PW}}\left(1 - \theta_s\right)' - Q_{\text{PW}}}{\ln(1 - \theta_s)} * H_s * n}{T}\right\} + \left\{\frac{\left(Q_{\text{PW}} - q_{\text{PW}}\right) * P_s * n}{T}\right\}
$$
\n
$$
TVI \tag{9}
$$

Where $(Q_{PW}-q_{PW})$ is the total deteriorating quantity of raw materials during period *t*.

3.4. The integrated inventory model

The integrated joint total cost *TC* for the buyer, the producer and the supplier is the sum of TC_B , TC_P and TC_S , and can be written as:

$$
TC = TC_B + TC_P + TC_S \tag{10}
$$

4. Numerical example

Let us consider the following numerical example: *T*=1 week. The buyer's parameter data: *D N*(12,000, 500²), *SL* = 90%, *LT* = 0.01, *A* = \$300, $F_B = 25 , $H_B = 15 , $P_B = 110 , $\theta_B = 0.08$, α_B = \$15, β_B = \$110. The producer's parameter data: $P = 24,000$ units, $S_P =$ \$500, $F_P =$ \$150, $F_{PW} =$ \$20, $H_P =$ \$12, H_{PW} = \$10, P_P = \$90, P_{PW} = \$85, θ_P = 0.095, $\theta_{PW} = 0.09$, $\beta_P = \$90$. The supplier's parameter data: $S = 250 , F_S $= $125, H_S = $8, P_S = $75, \theta_S = 0.1.$

Substituting the above parameter's data into the model, then the optimal solution can be derived as shown in Tables 1 and 2.

In Tables 1 and 2, for each viewpoint, the optimal lot size and delivery number is obtained where the total cost is minimized. The joint total cost from the integrated viewpoint is \$109,225, which results in an impressive joint total cost reduction compared with any independent viewpoint from the buyer, the producer or the supplier. This result suggests that the partners in the supply chain could develop a mutual agreement in order to benefit from the lowest joint total cost. Besides, in Table 2, because the demand is random, it is necessary to consider the safety stock to prevent stockout. The phenomenon results in the buyer's holding cost, deterioration cost, and overstock cost to be increased noticeably, hence the buyer's total cost TC_B is much more than the producer and the supplier's total cost.

5. Analysis of variance

 In order to study how various key factors affect the buyer's total cost, the producer's total cost, the supplier's total cost, and the joint total cost, a analysis of variance for key factors of service level, standard deviation, lead time, and deterioration rate is performed. The experiment is performed as shown in Table 3. Then we conduct an analysis of variance with statistical software SPSS, and the main result is summarized in Table 4.

The main purpose of this section is to study the relationship between the total costs (TC_B, TC_P, TC_S, TC) and factors (*SL*, σ , *LT*, θ). Table 4 indicates some conclusions from this analysis. In the analysis, we use the significance level, α = 5%. Thus, we see that the main factors of *SL*, σ , *LT* and θ significantly affect the total costs of TC_B , TC_P , TC_S and *TC*. Besides, in the factor interaction, we see that σ^*LT , $\sigma^*\theta$ and $LT^*\theta$ has a significance, which indicates significant interaction between these factors. However, *SL*LT* and *SL*θ* has a little significance, which indicates no significant interaction between these factors.

6. Conclusions

In a competitive environment, the cooperation is especially important in the supply chain. The result in this study has shown that the integrated viewpoint is more economical compared with an independent viewpoint.

This study has also performed an analysis of variance to show that how various key factors for the random demand affect various total costs for the partners in a supply chain. The key factors include service level, standard deviation, lead time, and deterioration rate. When the demand is random, in order to have a higher service level, the safety stock must be increased in order to prevent stockout; but sometime it turns into overstock. In this study, adopting the most economic service level is suggested, instead of the perfect service level, and this adoption can obtain the lowest joint total cost, buyer's total cost, producer's total cost, and supplier's total cost.

References

- Ben-Daya, M., Raouf, A., 1994. Inventory models involving lead time as decision variable. Journal of the Operational Research Society 45(5), 579-582.
- Chopra, S., and Meindl, P., 2001. Supply chain management. Printice-Hall, Inc., Upper Saddle River, New Jersey 07458, 179-257.
- Ghare, P.M., Schrader, G.F., 1963. A model for exponentially decaying inventory. The Journal of Industrial Engineering 14(5), 238-243.
- Kim, S.L., and Ha, D., 1997. Implementation of JIT purchasing: an integrated approach. Production Planning and Control 2, 152-157.
- Liao, C.J., Shyu, C.H., 1991. An analytical determination of lead time with normal demand. International Journal of Operations Production Management 11, 72-78.
- Moon, I., Choi, S., 1998. A note on lead time and distributional assumptions in continuous review inventory models. Computers and Operations Research 25(11), 1007-1012.
- Ouyang, L.Y., Yeh, N.C., Wu , K.S., 1996. Mixture inventory model with backorders and lost sales for variable lead time. Journal of the Operational Research Society 47, 829-832.
- Ouyang, L.Y., Chen, C.K., Chang, H.C., 2002. Quality improvement, setup cost and lead-time reductions in lot size reorder point models with an imperfect production process. Computer and Operations Research 29, 1701-1717.
- Rau, H., Wu, M.Y., Wee, H.M., 2003. Integrated inventory model for deteriorating items under a multi-echelon supply chain environment. International Journal Production Economics 86(2), 155-168.

Tersine, R.J., 1994. Principles of

inventory and materials management. Englewood Cliffs, New Jersey, 204-272.

- Wu, M.Y., Rau, H., Wee, H.M., 2003. Exponentially varying demand for integrated inventory model with varying production of deteriorating item. Journal of Statistics and Management Systems 6(2), 217-227.
- Yang, P.C., Wee, H.M., 2000. Economic order policy of deteriorated item for vendor and buyer: an integrated approach. Production Planning and Control 11(5), 474-480.

Fig. 1. The supply chain system

Fig .2. Inventory level of the supply chain

Viewpoint	n	$\boldsymbol{t}_{\boldsymbol{P}}$	SS	q_B	q _p	q_{PW}	$\varrho_{\scriptscriptstyle PW}$
Buyer		0.0878	253	2,099	2,099	2,116	2,148
Producer	14	0.0486	184	1,164	1,164	1,169	1,178
Supplier	42	0.0219	118	524	524	525	527
Integrated	13	0.0517	190	1,237	1,237	1,243	1,253

Table 1 Quantity for various viewpoints

Viewpoint	n	TC_B	TC_P	TC_{S}	TC
Buyer		58,381	29,429	35,028	122,838
Producer	14	64,189	24,547	20,666	109,403
Supplier	42	105,309	37,345	13,864	156,519
Integrated	13	62,900	24,609	21,716	109,225

Table 2 Total cost for various viewpoints

Table 3 The related data for experiment result

SL	$\pmb{\sigma}$	LT	$\boldsymbol{\theta}$	TC_B	TC_P	TC_S	TC
50%	$\boldsymbol{0}$	0.01	$-4%$	8,947	20,621	11,651	41,219
			4%	11,521	25,304	15,071	51,896
		0.05	$-4%$	24,193	30,371	18,996	73,560
			4%	34,777	40,392	26,817	101,985
	2,000	0.01	$-4%$	235,405	28,431	40,979	304,816
			4%	266,572	34,448	51,625	352,645
		0.05	$-4%$	237,202	44,341	60,435	341,978
			4%	280,559	54,326	74,380	409,265
	$\boldsymbol{0}$	0.01	$-4%$	8,947	20,621	11,651	41,219
			4%	11,521	25,304	15,071	51,896
		0.05	$-4%$	24,193	30,371	18,996	73,560
70%			4%	34,777	40,392	26,817	101,985
	2,000	0.01	$-4%$	168,232	33,566	39,787	241,585
			4%	197,371	39,672	46,386	283,429
		0.05	$-4%$	178,130	49,670	55,158	282,958
			4%	220,537	61,416	63,888	345,840
90%	$\boldsymbol{0}$	0.01	$-4%$	8,947	20,621	11,651	41,219
			4%	11,521	25,304	15,071	51,896
		0.05	$-4%$	24,193	30,371	18,996	73,560
			4%	34,777	40,392	26,817	101,985
	2,000	0.01	$-4%$	149,204	44,520	42,090	235,814
			4%	173,411	56,586	52,457	282,455
		0.05	$-4%$	162,776	64,239	56,190	283,205
			4%	199,932	86,625	70,630	357,187

Table 4

	Dependent		Degree			
Factor	variable	Sum of square	of	Mean square	F test	Significance
			freedom			
\overline{SL}	TC_B	7638746669.08	$\overline{2}$	3819373334.54	574.87	.000
	TC_P	552971599.00	$\frac{2}{2}$	276485799.50	54.00	.000
	TC_S T _C	32926025.33 5313122763.08		16463012.67 2656561381.54	9.19 279.60	.007
		207393202262.04	$\overline{1}$	207393202262.04	31215.53	.000 .000
σ	TC_B	2558039424.00	1	2558039424.00	499.56	.000
	TC_P TC_S	7935206666.67	1	7935206666.67	4427.29	.000
	TC	354098897867.04	$\mathbf{1}$	354098897867.04	37268.98	.000
LT	TC_B	1741607325.38	1	1741607325.38	262.14	.000
	TC_P	1631982352.67	1	1631982352.67	318.71	.000
	TC_S	1129293204.17	1	1129293204.17	630.07	.000
	TC	13394382768.38	1	13394382768.38	1409.76	.000
$\overline{\theta}$	TC_B	2540127777.04	1	2540127777.04	382.32	.000
	TC_P	526575280.17	1	526575280.17	102.84	.000
	TC_S	403850104.17	$\mathbf{1}$	403850104.17	225.32	.000
	TC	8731428685.04	$\mathbf{1}$	8731428685.04	918.99	.000
$\overline{SL^* \sigma}$	TC_B	7638746669.08	$\overline{2}$	3819373334.54	574.87	.000
	TC_P	552971599.00		276485799.50	54.00	.000
	TC_S	32926025.33		16463012.67	9.19	.007
	TC	5313122763.08	2222222222	2656561381.54	279.60	.000
SL^*LT	TC_B	39122196.75		19561098.38	2.94	.104
	TC_P	14218858.33		7109429.17	1.39	.298
	TC_S	7763420.33		3881710.17	2.17	.171
	TC	51648800.25		25824400.13	2.72	.119
$SL^*\theta$	TC_B	11907245.58		5953622.79	.90	.442
	TC_P	25807708.33		12903854.17	2.52	.135
	TC_S	7319324.33		3659662.17	2.04	.186
	TC	16291406.58		8145703.29	.86	.456
σ^*LT	TC_B	29404134.38	$\overline{1}$	29404134.38	4.43	.065
	TC_P	99552266.67	1	99552266.67	19.44	.002
	TC_S	104516960.67	1	104516960.67	58.31	.000
	TC	218400633.38	1	218400633.38	22.99	.001
$\overline{\sigma^*\theta}$	$\overline{TC_B}$	1175426070.04 24389568.17		1175426070.04 24389568.17	176.92 4.76	.000 057
	TC_P TC_S	40052000.67	1	40052000.67	22.35	.001
	TC	2074997470.04		2074997470.04	218.39	.000
$LT^*\theta$	TC_B	162432457.04		162432457.04	24.45	.001
	TC_P	53826140.17		53826140.17	10.51	.010
	TC_S	21481768.17		21481768.17	11.99	.007
	TC	610858690.04		610858690.04	64.29	.000
Error	TC_B	59795191.54	$\overline{9}$	6643910.17		
	TC_P	46085365.50	9	5120596.17		
	TC_S	16131050.00	9	1792338.89		
	TC	85510517.04	9	9501168.56		
Total	$\overline{TC_B}$	533903078249.00	24			
	TC_P	43524836546.00	24			
	TC_S	41385799554.00	24			
	TC	1243873266641.00	24			

The main result for analysis of variance

供應鏈的聯合總成本最小化。經由數學例題的結果證明: 由整合性的觀點所獲得 分析探討一些關鍵因子對於最佳解的影響。根據原計畫的預期目標分述如下: (1) (2) (3) (4) (1) : (2) : 最後,提供數個議題作為未來研究之方向: (1) (2) (vendor managed inventory, VMI) (3)

計畫成果自評**:**