行政院國家科學委員會專題研究計畫　成果報告

供應鏈夥伴關係間的損耗商品之整合性存貨模式

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行政院國家科學委員會專題研究計畫成果報告

供應鏈夥伴關係間的損耗商品之整合性存貨模式
Integrated inventory model for deteriorating item in the supply chain partnership

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摘要

近年來，由於企業環境競爭愈來愈激烈，有效的供應管理已成為非常重要的課題。本研究主要在探討供應鏈夥伴關係的損耗商品之整合性存貨模式，透過買賣雙方的夥伴關係發展最佳的存貨政策，以降低供應鏈聯合總成本。本研究透過數學模式建構買賣雙方之存貨模式，並應用電腦程式的撰寫，搜尋最佳解，目的在取得供應鏈中的聯合總成本最小化。藉由數學範例及敏感度分析的結果證明，在供應鏈中買賣雙方若採取整合性的夥伴關係策略，可獲得供應鏈聯合總成本最佳化的存貨政策。

關鍵字: 供應鏈管理、夥伴關係、損耗性商品、整合性存貨模式

ABSTRACT

In a competitive market environment, it is important to consider the integrated approach to inventory control in a supply chain. The objective of this research is to consider a deteriorating item in the supply chain to develop an optimal joint total cost from the perspectives of a single buyer and supplier. A computer code is developed to derive the optimal solution. Numerical examples and sensitivity analysis are given to validate the results. It can be shown that the integrated approach results in an impressive cost reduction compared with an independent decision by the buyer or the producer.

Key words: Supply chain management, partner relationship, deteriorating items, integrated inventory model.

1. Introduction

The research in deteriorating inventory is becoming more important, because decay and deterioration occur in products, such as fish, fruit and vegetables. Deterioration inventory models have been widely studied by several authors in recent years. Ghare and Schrader [5] were the first researchers to consider decaying inventory for a constant demand. Other authors such as Covert and Philip [3], Dave

[4], Elsayed and Teresi [5], Mak [15], Kang and Kim [9], and Heng et al.[8] continued to refine the model.

There are a number of studies that incorporate the perspectives of both the buyer and the supplier. Goyal [7] first introduced the idea of joint total cost for the buyer and the supplier. Chakravarty and Martin [1] derived the optimal order period and discount price with joint minimum cost for the buyer and the supplier. Kao [10] considered both the perspectives of buyer and the supplier for deteriorating inventory with a constant exponential rate. Susan et al. [18] considered the cooperative and the non-cooperative situation, and obtained an optimal order price and quantity for each case. Kim and Ha [11] developed an integrated inventory model with JIT concepts and small lot size to derive the minimal joint total cost. Chou [2] developed an integrated two-stated inventory model for deterioration items, his research results shows that the cooperative strategy of the buyer and the supplier results in higher profit.

Pan and Liao [16] implemented the JIT concept of small lot size to derive the optimal solution. Their research showed that the use of JIT concept is worth pursuing. Larson [14] modified the model by Pan and Liao [16] and considered the transportation cost, the receiving cost, and the quality control cost for small size deliveries. Kim and Ha [12] applied the small size deliveries concept from the perspectives of both the buyer and the supplier. Kim and Ha [12] extended the research to consider the perspectives of both the supplier and the buyer in a JIT environment. Later, Wee and Jong [19] applied the concept of multiple deliveries and developed an optimal deterioration production strategy. Lin [13] considered the perspectives of both the buyer and supplier, and applied a single order and multiple deliveries in model development. Wu [20] extended the research to consider the deterioration inventory. Yang and Wee [21] developed an integrated deterioration inventory model for both buyer and vendor, but they simplified the problem formation by assuming continuous demand. In this paper, our model assumes discrete demand and deliveries.

2. Assumption and notation

The mathematical model developed in this paper is based on the following assumptions:

(1) Planning period is known.
(2) A single buyer and producer are assumed.
(3) Both demand rate and production rate are deterministic and constant.
(4) Production rate is greater than demand rate.
(5) Deterioration rate is deterministic and constant.

(6) Lead-time is assumed to be negligible.
(7) Shortage is not allowed.
(8) Multiple lot-size deliveries per order are considered instead of single delivery per order.
(9) The same lot-size delivered each time.

The following notations are used：

T	Planning period
T_P	Ordering cycle
t_1	Producer's starting time for each order cycle T_P
t_2	The period of production quantity q
N	Delivery times per planning period T
n	Delivery times per ordering cycle T_P
n_P	Delivery times from the producer to the buyer of production period
t_3	The period of the product from the point of n_P to the ending of producing
D	Demand rate on the buyer
q	Lot-size per delivery for the buyer
Q_B	Lot-size per order for the buyer, $Q_B=n*q$
$I_B(t')$	Inventory level at time t' for the buyer.
$I_{Pi}(t')$	Inventory level at time t' for the producer.
A_P	The producer's total inventory quantity during the period T_P
A	Ordering cost per order for the buyer
F	Delivery cost per deliver for the buyer
H_B	Holding cost per unit for the buyer
P_B	Deteriorating cost per unit for the buyer
θ_B	Rate of deterioration for the buyer
P	Production rate on the producer
S	Setup cost per setup for the producer
H_P	Holding cost per unit for the producer
P_P	Deteriorating cost per unit for the producer
θ_p	Rate of deterioration for the producer
TC_B	Total cost for the buyer.
TC_P	Total cost for the producer.
TC	Joint total cost for a single buyer and producer.

3. Model development

3.1 The Buyer's Inventory Model

The buyer inventory model for deteriorating item with multiple lot-size deliveries can be depicted as in Figure 1.

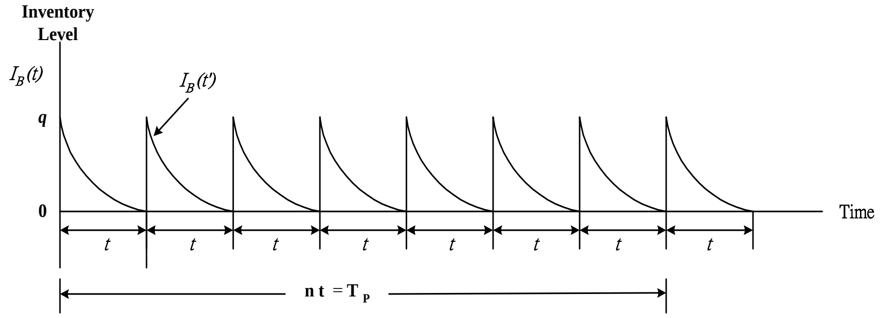

Figure 1. Inventory level of a single buyer

The inventory level at time *t'*, $I_B(t')$ can be expressed by the following equation

$$\frac{dI_B(t')}{dt'} = -D - \theta_B I(t') \qquad 0 \le t' \le t \qquad (1)$$

This differential equation can be solved as follows

$$I_B(t') = \frac{D}{\theta_B}\left(e^{\theta_B(t-t')} - 1\right) \qquad 0 \le t' \le t \qquad (2)$$

Where, the extreme points the inventory becomes $I_B(0) = \frac{D}{\theta_B}\left(e^{\theta_B t} - 1\right) = q$ and $I_B(t) = 0$. Inventory quantity on hold during the period *t* is

$$\int_0^t I_B(t')dt' = \int_0^t \frac{D}{\theta_B}\left(e^{\theta_B(t-t')} - 1\right)dt' = \frac{D}{\theta_B^2}e^{\theta_B t} - \frac{D + D\theta_B t}{\theta_B^2} \qquad (3)$$

The deteriorating cost during the period is $\left(\frac{D}{\theta_B}\left(e^{\theta_B t} - 1\right) - D * t\right) * P_B$ (4)

Then, the total cost for the buyer per *T* can be expressed as

$$TC_B = \frac{A}{T_P} * \frac{1}{T} + \frac{F * N}{T} + \frac{\left(\frac{De^{\theta_B t}}{\theta_B{}^2} - \frac{D + D\theta_B t}{\theta_B{}^2}\right) * H_B * N}{T} + \frac{\left(\frac{D}{\theta}\left(e^{\theta_B t} - 1\right) - D * t\right) * P_B * N}{T}$$

(5)

3.2 The Producer’s Inventory Model

The producer inventory model for deteriorating items with multiple lot-size deliveries can be depicted as in Figure 2.

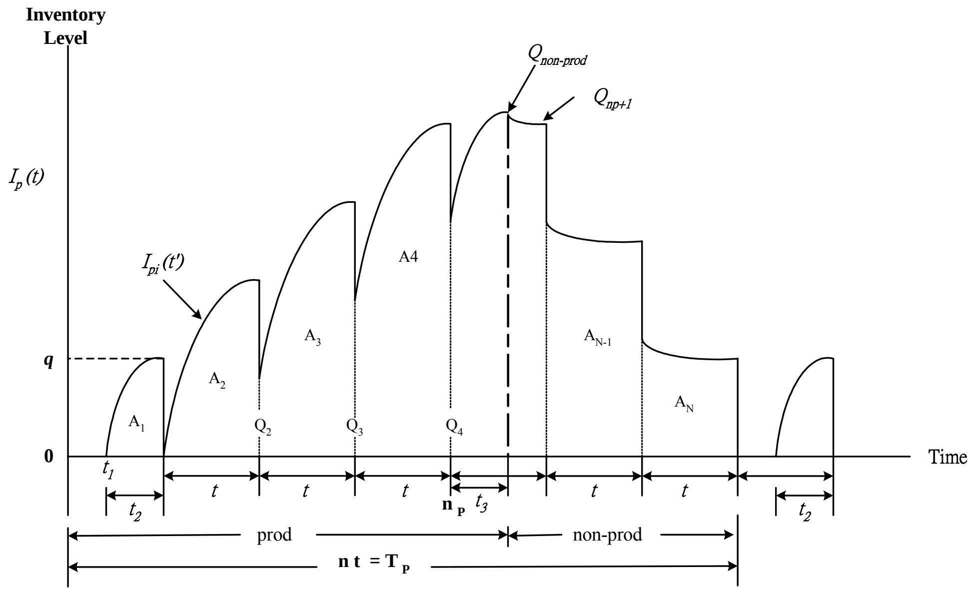

Figure 2 Inventory level of a single producer

The inventory level by the producer in the first period can be expressed by the following equation:

$$\frac{dI_P(t')}{dt'} = P - \theta_P I_P(t') \qquad 0 \le t' \le t_2,\ \ t_2 = t - t_1 \tag{6}$$

The differential equation is solved as follows

$$I_P(t') = \frac{P}{\theta_P}\left(1 - e^{(-\theta_P t')}\right) \qquad 0 \le t' \le t_2 \tag{7}$$

At the extreme points, $I_P(0) = 0$ and $I_P(t_2) = q$, one can derive

$$t_2 = -\frac{\ln\left(\dfrac{P - q\theta_P}{P}\right)}{\theta_P} = -\frac{\ln(P - q\theta_P)}{\theta_P} + \frac{\ln(P)}{\theta_P} \tag{8}$$

The inventory level for the *i*th time after the first time delivery is

$$\frac{dI_{Pi}(t')}{dt'} = P - \theta_P I_P(t') \qquad 0 \le t' \le t,\ \ 0 \le i \le n_P + 1 \tag{9}$$

Producer's opening inventory before the *i*th delivery is $I_{Pi}(0) = Q_{i-1}$. Solving the differential equation (9), one can derive the inventory level for the *i*th delivery as

$$I_{Pi}(t') = \frac{P}{\theta_P}\left(1 - e^{(-\theta_P t')}\right) + Q_{i-1}e^{\theta_P t} \qquad 0 \le t' \le t,\ \ 0 \le i \le n_P + 1 \tag{10}$$

From (10), one can derive the lot-size of the non-production period

$$Q_{non-prod(1)} = \frac{P}{\theta_P} + \frac{qe^{3\theta_P t}}{e^{\theta_P(t'+tn_P+t)}(e^{\theta_P t}-1)} - \frac{qe^{3\theta_P t}}{e^{\theta_P(t'+2t)}(e^{\theta_P t}-1)} - \frac{P}{\theta_P e^{\theta_P(tn_P-t+t')}} \tag{11}$$

According to Ghare and Schrader [6], the inventory becomes Q_{nP+1}, as derived from (11) as

$$Q_{nP+1} = Q_{non-prod}\left(1-\theta_P\right)^{t-t_3} \tag{12}$$

One can derive the ending inventory quantity for $(n_P + i)th$ time as

$$Q_{nP+i} = (Q_{nP+i-1} - q)\left(1-\theta_P\right)^t \qquad n_P + 1 \le i \le n \tag{13}$$

The producer's inventory quantity for the *n*th time is equal to the lot-size per delivery. From (11), (12) and (13), under the assumed conditions, can derive the lot size at non-production time $t = t_3$ as

$$Q_{non-prod(2)} = \frac{q\left((1-\theta_P)^{t'-nt+n_P t-t} - (1-\theta_P)^{t'-t}\right)}{(1-\theta_P)^{-t'} - 1} \tag{14}$$

Total cost for the producer per period T_P can be expressed as the sum of setup cost, holding cost, and deteriorated cost, i.e.

$$TC_P = \frac{S}{T_P} + \frac{\left(\dfrac{P(t_2 + t(n_P-1) + t_3) - nq}{\theta_P}\right) * H_P}{T_P} + \frac{\left(P(t_2 + t(n_P-1) + t_3) - nq\right) * P_P}{T_P} \tag{15}$$

3.3 The integrated joint total cost

The integrated joint total cost of the buyer and producer, *TC* is the sum of (5) and (15), .e. $TC = TC_B + TC_P$ (16)

4. Numerical example

Substituting the following parameters into the derived the buyer and producer inventory model for deteriorating items with multiple lot-size deliveries, a computer code is developed to derive the optimal solution. T =1 year, D = 12,000 units; A =\$25;

$F = \$25$; $H_B = \$12$; $P_B = \$100$; $\theta_B = 0.05$. $P = 24{,}000$ units; $S = \$500$; $H_P = \$10$; $P_P = \$95$; $\theta_P = 0.045$.

Figure 3 shows the comparison results among three views. Form Figure 3, one can derive the optimal delivery times *n* for different viewpoint.

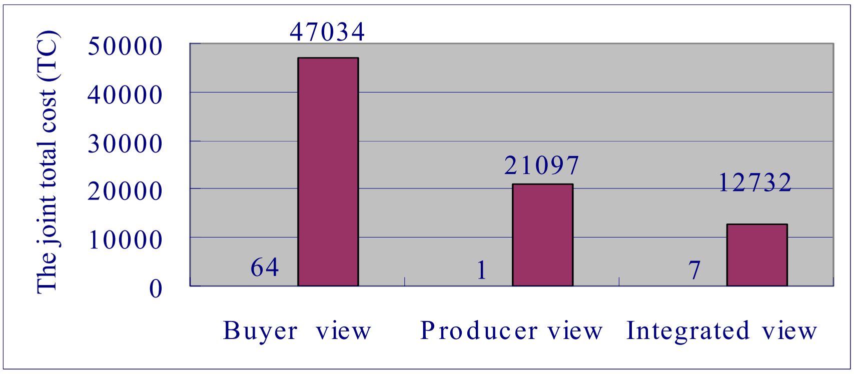

Figure 3 Comparison among the three views

The main results drawn from the numerical example are as follows: In Figure 3, the optimal decision using the integrated approach is 7 times delivery per order cycle and the joint total cost is \$12,732; the optimal solution from the buyer's view is 64 deliveries per order cycle, the joint total cost is \$47,034. It will incur an increase joint total cost of \$34,302(\$47,034-\$12,732). On the other hand, from the point of view form the producer, the optimal decision from the producer's view is 1 delivery per order cycle, the joint total cost is \$21,097. It will incur an increase joint total cost of \$8,365(\$21,097-\$12,732). In the above results, it can be shown that the integrated approach results in an impressive cost reduction compared with the independent decision by the buyer or the producer.

5. Sensitivity analysis

In order to study the how the various parameters effect the optimal solution of the integrated inventory model, a sensitivity analysis is performed. The value of each parameter is changed by ±10%、±20%、±30% for the joint total cost *TC*. The results of the sensitivity analysis of *TC* for all the parameters are shown in Table 1.

Table 1. Sensitivity of all the parameters to *TC*

	-30%	-20%	-10%	10%	20%	30%
θ_B	-1.12	-0.75	-0.37	0.37	0.74	1.11
θ_p	-3.54	-2.35	-1.17	1.16	2.31	3.44
D	-10.25	-6.06	-2.60	1.91	3.05	3.37

P	-14.27	-7.62	-3.18	2.45	4.25	5.75
A	-0.54	-0.36	-0.18	0.18	0.36	0.53
F	-4.07	-2.64	-1.26	1.24	2.45	3.53
H_B	-2.79	-1.79	-0.89	0.88	1.76	2.55
P_B	-1.12	-0.74	-0.37	0.37	0.74	1.10
S	-11.59	-7.48	-3.64	3.51	6.85	10.04
H_P	-8.33	-5.41	-2.66	2.59	5.11	7.54
P_P	-3.42	-2.02	-1.13	1.11	2.21	3.31

The values of the joint total cost TC are not sensitive to buyer's order cost A, slightly sensitive to θ_B、θ_p、F、H_B、P_B、H_P、P_P, and very sensitive to D and P.

6. Conclusion

In the past, most researches only consider the viewpoint of either the buyer or the producer. In a competitive market environment, it is important to consider the perspectives of both the buyer and the supplier. This study develops an integrated inventory model of a deteriorating item in the supply chain with multiple lot-size deliveries to derive an optimal joint total cost. A computer code is developed to derive the optimal solution. A numerical example and sensitivity analyses are given to validate the results. This study shows that the optimal policy adopted by the integrated approach results in consistent lower joint total cost.

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研究結果自評:

本研究主要在探討供應鏈夥伴關係的損耗商品之整合性存貨模式，透過買賣雙方的夥伴關係發展最佳的存貨政策，以降低供應鏈聯合總成本。由研究結果證明，在供應鏈中買賣雙方若採取整合性的夥伴關係策略，可獲得供應鏈聯合總成本最佳化的存貨政策。因此，可得知在目前的供應鏈競爭環境下，為了達到全面的最佳化，就必須採取整合性的觀點。

根據本研究計劃預計的目標可彙總如下:

(1) 發展製造商與零售商的損耗性商品之供應鏈存貨模式
(2) 應用電腦程式的撰寫求出供應鏈中製造商及零售商的聯合總成本
(3) 透過數學例題及敏感度分析證明藉由夥伴關係可達到買賣雙方整體系統成本的最小化

以上之研究目的都以遵照原計畫執行達成，本研究議題在理論及實務上都具相當重要的地位，以下將其重要性說明如下:

(1) 損耗性性商品(如: 魚肉、生鮮蔬果等)與生活習習相關，在探討相關類型的存貨模式時，應加入此因素予以探討。
(2) 在目前競爭如此激烈的供應鏈的環境下，已不能以自我的立場來考量，必須講求全面的最佳化，才能在競爭的環境下生存。
(3) 策略聯盟是由買賣雙方藉由合作的關係，達到風險共擔與利潤共享的境界。因此，成功的整合供應鏈夥伴，達到系統全面成本最佳化，讓供應鏈中所有的成員都能減少成本並增加利潤是相當重要的。

本研究已遵照原計畫執行完成，並將內容說明於正文中，透過正文的內容可知，本研究已達到原計畫書中所提出的預期目標與成果，本研究希望能把實際成果提供學術界及產業界參考。以下為本研究計畫對學術與產業界的貢獻:

(1) 對學術界之貢獻: 讓學界人士對於損耗性存貨模式的重視、瞭解整合性供應鏈模型之建構、瞭解供應鏈成員間夥伴關係合作的重要、瞭解損耗性商品模式在業界的應用。
(2) 對於產業界之貢獻: 使業界對損耗性商品在庫存管理上的重視、協助業界制定最佳的策略來處理損耗性商品、使業界瞭解供應鏈中所有成員間彼此合作的重要性、提供研究結果讓合適的產業分享，協助其改善及建立較佳的存貨管理系統。