Conservative schemes and degenerate scale problems in the null-field method for Dirichlet problems of Laplace's equation in circular domains with circular holes

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Abstract

Recently, the null-field method (NFM) has been proposed by Chen and his co-researchers {\bf for solving boundary value problems} involving circular domains with circular holes. The explicit algebraic equations of the NFM are derived in our recent paper \cite{LLHL2011}. However, even for the Dirichlet problem of Laplace's equation, when the logarithmic capacity (transfinite diameter) $C_{\Gamma } = 1$, the solutions may not exist, or not unique if existing, to cause a singularity of the discrete algebraic equations. {\bf The singularity of Dirichlet problems by the integral equations is first reported in Christiansen \cite{Ch75}, and then in \cite{CS2007,CLKC2012} called the degenerate scale problems.}

In this paper, the new conservative schemes of NFM are {\bf proposed}. The conservative schemes {\bf can} always bypass the degenerate scale problems; though numerically it causes a severe instability. A new pseudo-singularity property is discovered that only the minimal singular value $\sigma_{\min}$ of the discrete matrices is infinitesimal to cause the instability. To restore good stability of the conservative schemes, the over-determined systems and the truncated singular value decomposition (TSVD) are proposed. As a result, the over-determined systems are more advantageous than TSVD due to simpler algorithms and slightly better performances in error and stability. More importantly, such numerical techniques can also be used, to deal with all the degenerate scale problems of the original NFM in \cite{CLKC2001,CKL2002,CLCL2002} as well as the boundary element

method (BEM).

Keyword：Null-field method, conservative scheme, degenerate scale problem, pseudo-singularity, circular domains, overdetermined system, the truncated singular value decomposition.