Family of Invariant cantor sets as orbits of differential equations. II: Julia sets

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Abstract

The Julia set of the quadratic map $f_\mu(z) = \mu z(1 - z)$ for μ not belonging to the Mandelbrot set is hyperbolic, thus varies continuously. It follows that a continuous curve in the exterior of the Mandelbrot set induces a continuous family of Julia sets. The focus of this article is to show that this family can be obtained explicitly by solving the initial value problem of a system of infinitely coupled differential equations. A key point is that the required initial values can be obtained from the anti-integrable limit $\mu \to \infty$. The system of infinitely coupled differential equations reduces to a finitely coupled one if we are only concerned with some invariant finite subset of the Julia set. Therefore, it can be employed to find periodic orbits as well. We conduct numerical approximations to the Julia sets when parameter μ is located at the Misiurewicz points with external angle 1/2, 1/6, or 5/12. We approximate these Julia sets by their invariant finite subsets that are integrated along the reciprocal of corresponding external rays of the Mandelbrot set starting from the anti-integrable limit $\mu = \infty$. When μ is at the Misiurewicz point of angle 1/128, a 98-period orbit of prescribed itinerary obtained by this method is presented, without having to find a root of a 298-degree polynomial. The Julia sets (or their subsets) obtained are independent of integral curves, but in order to make sure that the integral curves are contained in the exterior of the Mandelbrot set, we use the external rays of the Mandelbrot set as integral curves. Two ways of obtaining the external rays are discussed, one based on the series expansion (the Jungreis-Ewing-Schober algorithm), the other based on Newton's method (the OTIS algorithm). We establish tables comparing the values of some Misiurewicz points of small denominators obtained by these two algorithms with the theoretical values.

Keyword：Julia set; Mandelbrot set; external ray; anti-integrable limit