Hybrid and Collocation Trefftz Methods for Traction Boundary Conditions in Linear Elastostatics

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Abstract

For linear elastostatics, the particular solutions and the fundamental solutions can be used for the
admissible functions, so that only the boundary conditions need to be satisfied, thus called the
boundary methods (or Trefftz methods). Hence, how to couple the boundary conditions is a crucial
issue. There are basically two types of boundary conditions: (I) the displacement (i.e., Dirichlet)
condition and (II) the traction (i.e., Neumann) condition. In this paper, the coupling techniques for the
traction (i.e., Neumann condition) are the main theme for boundary methods, because the traction
condition may be regarded as the basic condition and the displacement condition as the natural
condition. The Lagrange multiplier used for the displacement (i.e., Dirichlet) condition is well known in
mathematics community (see Babu∨ska, 1973 [1]; Babu∨ska et al., 1978 [2]; Li, 1998 [20]; Pitk¨aranta,
1979 [40]), but the Lagrange multiplier used for the traction (i.e., Neumann) condition is popular for
elasticity problems by the Trefftz method in engineering community, which is called the hybrid Trefftz
method (HTM) (see de Freitas, 1998 [7]; de Freitas and Ji, 1996 [8,9]; Jirousek, 1978 [12]; Jirousek and
Venkstesh, 1992 [13]; Jirousek and Wroblewski, 1996 [14]; Qin, 2000 [42]). On the other hand, the
collocation Trefftz method (CTM) can also be used to directly couple the

traction condition without
extra-multipliers. In this paper, a brief error analysis for the HTM is given, to provide the optimal
convergence rates. Numerical experiments of simple models are carried out to support this analysis.
The HTM has the merits: flexibility and robustness, so that it has been widely used in engineering
problems. Since the optimal convergence rates are the most important criterion in evaluation of
numerical methods, the global performance of the HTM is as good as that of the CTM, although the HTM
causes larger condition numbers and requires more CTU time for the computation of the simple models
in this paper. More numerical comparisons also show that using the particular solutions is more
advantageous than using the fundamental solutions in both HTM and CTM

Keyword：linear elastostatics, traction boundary conditions, Trefftz method, collocation Trefftz method, Lagrange multiplier