

Analyzing the Longitudinal Effect of Hypersonic Flow Past a Conical Cone
via the Perturbation Method

蔡博章, 邱永泰

Mechanical Engineering

Engineering

bjtsai@chu.edu.tw

Abstract

To analyze the hypersonic flow past a conical cone, the variations of gasdynamic properties subjected to the longitudinal curvature effect by using the perturbation method. An outer perturbation expansion has been carried out by recent researchers, but a problem occurred, the outer expansion solutions are not uniformly valid in the shock layer, however, the outcome near the conical body surface called vortical layer remains deflective. This study intends to discover uniformly valid analytical solutions in the shock layer by applying the inner perturbation expansions matching with the out expansions to analyze the characteristics in the whole region including shock layer and vortical layer. Starting from the zeroorder approximate solutions for hypersonic conical flow is then applied as the basic solutions for the outer perturbation expansions of a flow field. The governing equations and boundary conditions are also expanded via outer perturbations. Using an approximate analytical scheme in the derivation process, first-order perturbation equations can be simplified and the approximate closed-form solutions are obtained; furthermore, the various flow field quantities, including the normal force coefficient on the cone surface, have been calculated. According to the variations of gasdynamic properties, the longitudinal curvature effect for the hypersonic flow past a conical cone can be determined. Thicknesses of shock layer and vortical layer can be predicted as well. The physical phenomena inside both layers can be investigated carefully, the conditions for an elliptic cone with longitudinal curvature, $m = 1$ and $n = 2$ and

other conditions of parameters; the perturbation parameter, $\epsilon = 0.1$, semi-vertex angle of the unperturbed cone, $d = 10$, and hypersonic similarity parameter, $Kd = M_1 d = 1.0$, the thickness of vortical layer, δ_{VL} , can be calculated at the position angle of conical cone body, $\theta = 30^\circ$ was demonstrated here. Results show how very thin the vortical layer is approximately only 10% of the shock layer close to the body, the pressure in the whole shock layer is verified to be uniformly valid which agrees with previous studies. Large gradient changes in entropy and density are found when the flow approaches the cone surface, the most important is, this method provides a benchmark solution to the hypersonic flow past a conical cone and to assist the grids and numerics for numerical computation should be fashioned to accommodate the whole flow field region including the vortical layer of rapid adjustment, and let the analysis become more effective and low cost.

Keyword : Hypersonic flow; Perturbation method; Conical cone; Longitudinal curvature effect